F. H. Frisch sc: Berol:

THEORIA
MOTUUM PLANETARUM
ET
COMETARUM.

CONTINENS

METHODUM FACILEM EX ALIQUOT OBSERVATIONIBUS ORBITAS CUM PLANETARUM TUM COMETARUM DETERMINANDI.

UNA CUM CALCULO, QUO COMETÆ, QUI ANNIS 1680. ET 1681. ITEMQUE EJUS, QUI NUPER EST VISUS, MOTVS VERUS INVESTIGATUR.

AUCTORE LEONHARDO EULERO.

Sander W

1845 $\frac{28}{6}$.

Berolini Sumtibus AMBROSII HAUDE.
Bibliop. Reg. & Acad. Scient. privilegiati. 1745

De Motu planetarum & cometarum circa ſolem motorum.

I.

Si leges motus, quas planetæ primarii atque cometæ in curſu ſuo circa ſolem obſervant, conſulamus, non ſolum ellipſes, ſed etiam omnis generis ſectiones conicæ orbitas cometarum repræſentare poſſunt. Neque enim aliud diſcrimen inter planetas & cometas intercedit, niſi orbitæ figura, quæ ſi fuerit ellipſis non multum a circulo recedens, ſidus in ea motum planetæ nomen obtinuit, ſin autem orbita vehementer a circulo abhorreat, ſive ſit ellipſis admodum oblonga, ſive parabola, ſive adeo hyperbola, ejusmodi ſidera cometæ appellantur. Utrique autem, planetæ ſcilicet & cometæ, in curſu ſuo easdem ſequi leges motus merito ſtatuuntur, ſecundum quas tempora, quibus data ſpatia in orbitis ſuis abſolvunt, ſint in ratione compoſita arearum circum ſolem deſcriptarum directa, & reciproca ſubdu-

plicata laterum rectorum, seu ut vocari alias solent, parametrorum orbitarum: Sol autem in altero foco cujusque orbitæ fixus æstimatur. Ex hac autem lege non solum planetarum cometarumve motus in orbibus sive ellipticis sive parabolicis sive hyperbolicis definiri, sed etiam hæ ipsæ orbitæ per observationes aliquot determinari possunt.

2. Ex isto etiam fonte derivatæ sunt tabulæ astronomicæ motuum planetarum, ope quarum ex dato tempore seu anomalia media inveniri solet anomalia vera seu coæquata, & vicissim; quæ methodus vero tantum ad orbitas ellipticas circulo vicinas est accommodata. Quare si eandem theoriam in sensu latissimo stabilire atque ad motus etiam cometarum, qui in hyperbolis incedant, transferre velimus, ab instituto hoc consueto, quo relatio inter anomaliam veram & mediam definiri solet, recedere, alioque modo cursum ad calculum revocare oportebit. Primum enim incommodum methodi receptæ in hoc consistit, quod anomaliæ ab aphelio computentur: aphelium autem solæ orbitæ ellipticæ admittunt, parabolicæ autem & hyperbolicæ eo penitus carent. Quamobrem huic incommodo medela afferetur, anomalias a perihelio, quod in omnis generis orbitas æque competit, computando. Deinde etiam anomalia media uti non licebit, quod a tempore periodico pendeat, orbitæ autem parabolicæ & hyperbolicæ tempore periodico destituantur. Loco anomaliæ mediæ igitur ipsum tempus a momento, quo sidus in perihelio versatur, computatum adhibebo; anomalia vera autem erit distantia sideris heliocentrica a perihelio. His

igitur præmissis sequentia problemata evolvam, ex quibus natura motuum coelestium cum theoretice cognosci, tum etiam usus in praxi dilucide perspici poterit.

Problema I.

3. Data area, quam planeta vel cometa dato tempore circa solem descripsit, invenire latus rectum orbitæ.

Solutio.

Quoniam hic quantitates diversi generis, areæ scilicet & tempora inter se conferuntur, utrumque genus secundum certam constantemque mensuram exprimi debebit. Posita igitur distantia Solis a terra media = 100000, in ejusmodi unitatibus cunctas magnitudines exprimemus; atque adeo latus rectum orbitæ, quod quærimus, ex hac unitate definiri oportet. Aream autem circa solem descriptam per eandem mensuram in ejusmodi partibus quadratis, quarum 100000 semiaxem orbis magni constituunt, exhiberi pono. Sit igitur A area, quam planeta cometave circa solem descripsit, ad mensuram assumtam revocata; atque b denotet semissem lateris recti, seu applicatam ad axem transversum in foco normalem, in eadem mensura definiendam. Tempus denique posthac perpetuo exprimamus in diebus naturalibus temporis medii, & fractionibus diei decimalibus: sitque tempus propositum, quo area A circa solem confecta perhibetur, = T diebus; hoc igitur modo quantitates A, b, & T, quarum in hoc problemate mentio fit, ad numeros absolutos reducentur. Cum igitur ex natura motus corporum coelestium circa solem motorum sit tempus

T proportionale areæ A divisæ per $\sqrt{b}$, fiet $\frac{A}{T\sqrt{b}}$ = numero cuidam constanti, qui sit = m; ita ut sit T = $\frac{A}{m\sqrt{b}}$ seu $\sqrt{b} = \frac{A}{mT}$, & $b = \frac{AA}{mmTT}$; dummodo ergo iste numerus m fuerit cognitus, problema erit resolutum, eò quod semilatus rectum b reperitur expressum in ejusmodi partibus, quarum 100000 distantiam solis à terra mediam conficiunt. Ad numerum itaque m definiendum casum jam cognitum evolvamus. Cum scilicet constet, terram circa solem in orbita sua circumferri tempore anni siderei seu $365^d, 6^h, 8', 30''$; si ponamus T = 365, 256, & A = areæ totius orbitæ terrestris & b = semilateri recto orbitæ terræ, dabit fractio $\frac{A}{T\sqrt{b}}$ verum valorem numeri m. Sit ergo semiaxis transversus orbitæ terræ = c, quem sumsimus = 100000, erit semiaxis conjugatus = $\sqrt{bc}$. Denotet 1: π rationem diametri ad peripheriam, ita ut sit π = 3, 14159265, erit area circuli radio c descripti = πcc, quæ erit ad aream orbitæ terræ ut c ad $\sqrt{bc}$, unde fit area orbitæ terrestris A = $\pi c \sqrt{bc}$, ideoque $m = \frac{A}{T\sqrt{b}} = \frac{\pi c\sqrt{c}}{T}$ seu numerus m dabitur per numeros cognitos π = 3, 14159265; c = 100000, & T = 365, 256. Hinc per logarithmos valor numeri m reperietur:

$l\,\pi$	=	0, 4 9 7 1 4 9 8 7 2 7
$lc\sqrt{c}$	=	7, 5 0 0 0 0 0 0 0 0 0
		7, 9 9 7 1 4 9 8 7 2 7
subtr. lT	=	2, 5 6 2 5 9 7 3 5 8 8
erit $l\,m$	=	5, 4 3 4 5 5 2 5 1 3 9
ideoque m	=	2 7 1 9 8 9, 7 3 5

Inven-

Invento ergo numero $m = 271989,735$, erit orbitæ cujuscunque circa ſolem deſcriptæ latus rectum $2b = \frac{2AA}{mmTT}$.
Q. E. J.

Coroll. 1.

4. Si igitur cognita fuerit area, quam cometa dato tempore circa ſolem conficit, hinc latus rectum orbitæ, in qua movetur cometa, reperitur ita, ut ejus ratio ad diſtantiam Solis a terra mediam exhibeatur.

Coroll. 2.

5. Sin autem latus rectum orbitæ, quod ſumimus $= 2b$ jam fuerit cognitum, tempus aſſignari poterit, quo data orbitæ area circa ſolem abſolvitur. Sit enim area hæc deſcripta $= A$ erit tempus $T = \frac{A}{m\sqrt{b}}$ dier: ſiquidem, quod ubique eſt tenendum, longitudines in ejusmodi partibus exprimantur, quarum 100000 ſemiaxem transverſum orbis magni conſtituunt, & ſit $m = 271989,735$.

Coroll. 3.

6. Viciſſim ergo ex dato latere recto orbitæ $2b$ area aſſignari poteſt, quam planeta vel cometa dato tempore T circa ſolem deſcribit: tempore enim T in diebus expreſſo erit area deſcripta $A = mT\sqrt{b}$.

Problema II.

7. Datis ſectionis conicæ ABM diſtantia verticis a foco AS una cum latere recto, cujus ſemiſſis eſt BS, invenire Fig. 1.

 rela-

relationem, quæ inter diſtantiam cujusvis puncti M a foco S & anomaliam veram ſeu angulum ASM intercedit.

Solutio.

Sit diſtantia verticis a foco AS, ſeu pro orbitis planetarum cometarumve diſtantia perihelii A à ſole S nempe AS $= a$, & ſemiſſis lateris recti, nempe applicata BS $= b$. Dabit ergo recta AS producta axem ſectionis conicæ, ad quem ex puncto M demittatur perpendiculum MP; erit ex natura ſectionum conicarum diſtantia MS $= a + \frac{(b-a)}{a}.AP$. Cum autem ſit AP $= a +$ PS; ſi ponatur diſtantia MS $= y$ & angulus ſeu anomalia vera ASM $= v$, erit poſito ſinu toto $= 1$, $\cos v = \frac{-PS}{MS} = \frac{-PS}{y}$, ideoque PS $= - y \cos v$, & AP$= a-y \cos v$. Hoc ergo valore ſubſtituto fiet MS$=y=$ $a + \frac{(b-a)}{a}(a-y\cos v) = b - \frac{(b-a)y\cos v}{a}$. Hinc igitur prodit $\cos v = \frac{a(b-y)}{y(b-a)}$ & $y = \frac{a\ b}{a+(b-a)\cos v}$. Ex data ergo anomalia vera v reperitur diſtantia planetæ ſeu cometæ a ſole y; & viciſſim ex cognita hac diſtantia definitur anomalia vera. Q. E. J.

Coroll. 1.

8. Si anomalia vera ASM evaneſcit, ſitque $v = o$, erit $\cos v = 1$. hocque caſu prodit diſtantia a ſole $y = a$; nempe puncto M in A incidente ſit MS (y) $=$ AS (a). Simili modo ſi anomalia vera ASM $= v$ fiat angulus rectus, erit

$\cos v$

$\cos v = 0$, & fiet $y = b$; id quod perſpicuum eſt, quoniam punctum M in B incidit.

Coroll. 2.

9. Quod ſi ponamus $v = 180°$, exhibebit y diſtantiam aphelii a ſole; erit autem ob $\cos v = -1$, hæc diſtantia $= \frac{ab}{2a-b}$; ad quam ſi addatur diſtantia perihelii a ſole $= a$ prodibit axis transverſus orbitæ $= \frac{2aa}{2a-b}$; & ſemiaxis transverſus $= \frac{aa}{2a-b}$, unde diſtantia foci a centro orbitæ erit $= \frac{a(b-a)}{2a-b}$; & excentricitas $= \frac{b-a}{a}$.

Coroll. 3.

10. Si igitur fuerit $b = a$, ſectio conica erit circulus, centrum in S & radium AS $= a$ habens. Sin autem ſit $b > a$, curva erit ellipſis, quoad b fiat $= 2a$, quo caſu curva abit in parabolam ob axem transverſum $\frac{2aa}{2a-b}$ infinitum. At ſi fiat $b > 2a$, tum axis transverſus fit negativus, quo indicatur, curvam eſſe hyperbolam.

Coroll. 4.

11. Si fuerit $b < a$, curva ſemper erit ellipſis, ſed punctum A erit vertex a foco S remotior, ideoque aphelium repræſentabit; punctum ergo orbitæ e diametro oppoſitum

erit perihelium, cujus a foco S diſtantia erit $= \frac{ab}{2a-b}$, quæ minor eſt quam a ſi fuerit $b < a$.

Problema III.

Fig. 2. 12. Si dentur duæ cometæ planetæve a ſole diſtantiæ FS & GS una cum angulo FSG intercepto; atque inſuper cognitum ſit latus rectum orbitæ; determinare ipſam orbitam AFG.

Solutio.

Sit ſemilatus rectum $= b$, ac ponatur diſtantia FS $= f$ GS $= g$, atque angulus FSG $= \phi$; quæ ſunt data: ex quæſitis autem ſit diſtantia perihelii a ſole AS $= a$, & anomalia ſeu angulus ASF $= v$; erit angulus ASG $= v + \phi$. His poſitis problema præcedens duas nobis ſuppeditabit æquationes; SF $= f = \frac{ab}{a + (b-a)\cos v}$ & SG $= g$ $= \frac{ab}{a+(b-a)\cos(v+\phi)}$, ex quibus pro a duplex oritur valor $a = \frac{bf\cos v}{b-f+f\cos v} = \frac{bg\cos(v+\phi)}{b-g+g\cos(v+\phi)}$; qui inter ſe æquati dant $(b-g)f\cos v = (b-f)g\cos(v+\phi) = (b-f)$ $g(\cos v.\cos\phi - \sin v.\sin\phi)$, ideoque $\frac{(b-g)f}{(b-f)g} = \cos\phi -$ $\tan v.\sin\phi$, ex qua æquatione determinabitur anomalia v per quantitates mere cognitas, ſcilicet tang $v = \cot\phi -$

$(b-g)$

$\frac{(b-g)f}{(b-f)g \sin \phi}$ seu tang $v = \frac{(b-f)g \cos\phi - (b-g)f}{(b-f)g \sin\phi}$. Invento autem angulo ASF $= v$, distantia perihelii a sole erit AS $= a = \frac{bf \cos v}{b-f+f\cos v}$. Ex datis vero AS $= a$ & semilatere recto $= b$ orbita AFG definitur per probl. præc. Q. E. J.

Coroll. 1.

13. Si igitur detur area FSG una cum tempore, quo planeta seu cometa spatium FG percurrit, per problema primum reperitur latus rectum, ac proinde ex hoc problemate definietur ipsa orbita AFG.

Coroll. 2.

14. Primo scilicet ex datis semilatere recto b & distantiis FS $= f$, GS $= g$ una cum angulo FSG $= \phi$ cognoscetur positio axis AS ex angulo ASF $= v$, cum sit tang $v = \cot\phi - \frac{(b-g)f}{(b-f)g \sin\phi}$; & cognito angulo v erit AS $= a = \frac{bf\cos v}{b-f+f\cos v}$.

Coroll. 3.

15. Ex natura autem sectionum conicarum patet, si fuerit A orbitæ perihelium, rectarum FS & GS illam, quæ sit brevior, propiorem esse perihelio A; Quare cum sæpenumero incertum sit, utra rectarum FS & GS designari debeat littera f, breviorem semper hac littera insigniri conveniet.

Coroll. 4.

16. Si tangens anomaliæ v prodeat negativa, indicio hoc erit vel angulum ASF esse recto majorem (duobus tamen

rectis minorem) vel esse negativum, atque perihelium A intra loca F & G cadere. Cum igitur positio perihelii A sit anceps, & in duo loca, e diametro opposita incidat, ea erit vera, quæ breviori distantiæ f erit propior, altera vero positio dabit aphelium.

Problema IV.

Fig. 3. 17. Data orbita planetæ seu cometæ AM, invenire tempus, quo anomalia vera quævis ASM absolvitur.

Solutio.

Sit distantia perihelii a sole AS $= a$, semilatus rectum $= b$; & anomalia vera proposita ASM $= v$, atque distantia SM $= y$, erit $y = \frac{ab}{a + (b-a)\cos v}$, seu $\cos v = \frac{a(b-y)}{y(b-a)}$. Sit insuper area ASM $=$ A; erit per probl. 1. tempus quæsitum, quo planeta vel cometa spatium AM conficit $= \frac{A}{m\sqrt{b}}$ dierum. Quare ad tempus hoc definiendum aream ASM definiri oportet. Capiatur elementum Mm, & ducta Sm erit angulus MSm $= dv$, ideoque area trianguli MSm $= \frac{1}{2} yy\,dv$, quæ cum sit differentiale areæ ASM $=$ A, erit $dA = \frac{1}{2} yydv$, hincque A $= \frac{1}{2} \int yy\,dv$, & si loco y ejus valor in v substituatur, erit A $= \frac{1}{2} \int \frac{aabb\,dv}{(a + (b-a)\cos v)^2}$ quæ formula ut integrari queat, ponatur tang $\frac{1}{2}$ ASM $=$ tang $\frac{1}{2} v = t$, erit $dv = \frac{2dt}{1+tt}$; & $\cos v = \frac{1-tt}{1+tt}$, his substitutis fit A $=$

$\int aa$

$\int \frac{aabbdt\,(1+tt)}{(b+(2a-b)tt)^2}$. Sit $A = \frac{\alpha\, t}{b+(2a-b)tt} + \beta \int \frac{dt}{b+(2a-b)tt}$, atque comparatione instituta erit $\alpha + \beta = aab$, & $\beta - \alpha = \frac{aabb}{2a-b}$ hincque $\alpha = -\frac{aab(b-a)}{2a-b}$ & $\beta = \frac{a^3 b}{2a-b}$, ex quibus colligitur area $A = \frac{a^3 b}{(2a-b)} \int \frac{dt}{b+(2a-b)tt} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$. Hujus integralis constituendi sunt quatuor casus, quorum primus est:

I. Si $b = a$, quo curva fit circulus; hic casus promtissime ex prima formula resolvitur, qui dat $A = \frac{1}{2} \int aadv = \frac{1}{2} aav$; vel si hoc integrale per t expressum desideretur ob $v = 2$ A tang. t, fiet area $A = aa$ A tang. t.

II. Sit $b > a$ ita tamen ut sit $b < 2a$, quo casu orbita erit ellipsis, fitque ut vidimus $A = \frac{a^3 b}{2a-b} \int \frac{dt}{b+(2a-b)tt} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$. Est vero $\int \frac{dt}{b+(2a-b)tt} = \frac{1}{\sqrt{b(2a-b)}}$ A tang $t \sqrt{\frac{2a-b}{b}} = \frac{1}{\sqrt{b(2a-b)}}$ A sin $\frac{t\sqrt{(2a-b)}}{\sqrt{(b+(2a-b)tt}}, = \frac{1}{\sqrt{b(2a-b)}}$ A cos $\frac{\sqrt{b}}{\sqrt{(b+(2a-b)tt)}}$

Hinc fit $\int \frac{dt}{b+(2a-b)tt} = \frac{1}{2\sqrt{b(2a-b)}}$ A sin $\frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}$. sequentes ergo prodibunt expressiones aream A exhibentes

A =

$$A = \frac{a^3 \sqrt{b}}{(2a-b)\sqrt{(2a-b)}} \text{ A tang } t\sqrt{\frac{2a-b}{b}} - \frac{aab(b-a)t}{(2a-b)(b+(2a-b)tt)}$$

$$\text{vel } A = \frac{a^3 \sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \text{ A sin } \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} - \frac{aa(b-a)\sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} .$$

$$\frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} \text{ seu } A = \frac{a^3 \sqrt{b}}{2(2a-b)\sqrt{(2a-b)}} \left(\text{A sin } \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt} - \frac{(b-a)}{a} \cdot \frac{2t\sqrt{b(2a-b)}}{b+(2a-b)tt}\right)$$

III. Sit $b = 2a$, erit orbita parabola, atque erit ex priori æquatione $A = \int aadt(1+tt) = aa(t+\frac{1}{3}t^3)$, qui est unicus casus, quo area algebraice potest exhiberi.

IV. Sit $b > 2a$ erit orbita hyperbolica, fietque $A =$

$$\frac{aab(b-a)t}{(b-2a)(b-(b-2a)tt)} - \frac{a^3 b}{b-2a} \int \frac{dt}{b-(b-2a)tt}.$$ At hæc integratio a logarithmis pendet, fitque $\int \frac{dt}{b-(b-2a)tt} = \frac{1}{2\sqrt{b(b-2a)}}$

$$l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}.$$ Ex his ergo reperitur area quæsita $A =$

$$\frac{aab(b-a)t}{(b-2a)(b-(b-2a)tt)} - \frac{a^3\sqrt{b}}{2(b-2a)\sqrt{(b-2a)}}\, l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}$$ sive

$$A = \frac{a^3 \sqrt{b}}{2(b-2a)\sqrt{(b-2a)}}\left(\frac{b-a}{a} \cdot \frac{2t\sqrt{b(b-2a)}}{b-(b-2a)tt} - l\frac{\sqrt{b}+t\sqrt{(b-2a)}}{\sqrt{b}-t\sqrt{(b-2a)}}\right)$$

His igitur singulis casibus si reperta fuerit area $ASM = A$, erit tempus, quo ea absolvitur $= \frac{A}{m\sqrt{b}}$, quæ expressio tempus in diebus exhibebit, si fuerit $m = 271989, 735$ & A & b ea mensura definiantur, quam indicavi. Q. E. J.

Coroll.

Corol. 1.

18. Si ergo orbita fuerit circulus, erit tempus, quo angulus ASM $= v$ abſolvetur $= \frac{a v \sqrt{a}}{2m}$ ob $b = a$: quod quidem ex motu uniformi ſponte liquet. Ceterum quia in circulo perihelium non datur, anomaliæ veræ nomen improprie adhibetur.

Coroll. 2.

19. Si orbita fuerit ellipſis, quo area A commodius exprimatur, quæratur angulus ω, ut ſit tang. $\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang. $\frac{1}{2} v = \frac{t\sqrt{(2a-b)}}{\sqrt{b}}$. Angulo autem hoc ω invento, erit area A $= \frac{a^3 \sqrt{b}}{2(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\,\text{ſin}\,\omega\right)$, ideoque tempus per AM $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\,\text{ſin}\,\omega\right)$.

Coroll. 3.

20. Si orbita fuerit hyperbola, quæratur pariter angulus ω, ut ſit tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2} v$, ſeu $t = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ tang $\frac{1}{2}\omega$. Quo angulo ω invento erit area ASM $=$ A $=$ $\frac{a^3\sqrt{b}}{2(b-2a)^{\frac{3}{2}}}\left(\frac{(b-a)}{a}\,\text{tang}\,\omega - l\,\text{tang}\,(45^\circ + \tfrac{1}{2}\omega)\right)$, hincque tempus per arcum AM erit $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}\left(\frac{(b-a)}{a}\,\text{tang}\,\omega - l\,\text{tang.}\,(45^\circ + \tfrac{1}{2}\omega.\right)$

Coroll. 4.

21. Cum autem hic logarithmus tangentis anguli $45° + \frac{1}{2}\omega$ per integrationem sit inventus, perspicuum est, eum ex canone logarithmorum hyperbolicorum esse depromendum, posito sinu toto $= 1$. Hujusmodi vero canone deficiente, sumatur ex tabulis vulgaribus logarithmus tang $(45° + \frac{1}{2}\omega)$, & subtracto 10 ab ejus characterista, residuum, multiplicetur per numerum 2, 302585092994, atque productum erit log. hyperb. tang $(45° + \frac{1}{2}\omega)$; ubi notari convenit, hujus numeri 2, 30258509 &c. logarithmum esse $=$ 0, 3622156886.

Scholion 1.

22. Eadem expressio logarithmica l tang $(45° + \frac{1}{2}\omega)$ reperitur in hydrographia, si quæratur in mappa Nautica ad mentem Mercatoris construenda latitudo crescens respondens latitudini ω in superficie terræ. Quare ex tabulis nauticis, quibus latitudines crescentes (Gall. les latitudes croissantes) ad singulos gradus exhiberi solent, hi valores l tang $(45° + \omega)$ sumi poterunt. Interim tamen sine his tabulis calculus satis expedite instituetur hoc modo. Sit id quod quæritur nempe l tang $(45° + \frac{1}{2}\omega)$ $= x$. Tum in tabulis logarithmorum tangentium consuetis quæratur log. tang $(45° + \frac{1}{2}\omega)$, a quo auferatur log. tang 45° seu 10, 00000, residuum ponatur $=$ R, eritque $x = 2$, 302585092994. R, ideoque sumendis logarithmis erit $lx = lR +$ 0, 3622156886; unde ope tabulæ logarithmorum solitorum facile valor ipsius $x = l$ tang $(45° + \frac{1}{2}\omega)$ reperitur. Sit exempli gratia angulus $\omega = 37°, 22', 40''$. erit $\frac{1}{2}\omega =$

18°,

$18°, 41', 20''$ & $45° + \frac{1}{2}\omega = 63°, 41', 20''$. Tum ex tabulis erit.

l tang $(45° + \frac{1}{2}\omega)$	=	10, 3058582068
subtr. l tang 45°	=	10, 0000000000
erit R	=	0, 3058582068
ideoque lR	=	9, 4855201380
addatur	=	0, 3622156886
erit lx	=	9, 8477358266
unde obtinetur x	=	0, 7042645474

Scholion. 2.

23. Circa tempus in ellipsi assignandum notari oportet arcum seu angulum ω non more solito in gradibus & minutis, sed in partibus radii, qui ponitur $= 1$, exprimi debere. Scilicet si ω esset 180°, tum pro ω substitui deberet longitudo semiperipheriæ 3, 1415926535. Hinc igitur valor cujusvis anguli ω in partibus decimalibus radii 1 exprimi poterit; convertatur arcus ω in minuta secunda, sitque $\omega = n''$, atque manifestum est, hanc institui debere proportionem ob $180° = 648000''$, ut sit $648000 : 3, 1415926535 = n$: valorem ω, eritque ideo

$$\text{valor } \omega = \frac{3,\,1415926535}{648000}\,n. \quad \text{Cum}$$

ergo sit l 648000	=	5, 8115750059
subtr, l 3, 14159 &c.	=	0, 4971498727
a log. n subtrahatur	– – –	5, 3144251332

& residuum erit logarithmus ipsius ω in partibus decimalibus radii expressi. Vel ad logarithmum numeri n addatur constanter 4, 6855748668, eritque summa, postquam ejus characteristica fuerit denario minuta, logarithmus valoris quæsiti anguli ω.

Problema V.

Fig. 2. 24. Datis duabus a Sole diſtantiis FS & GS una cum angulo ad ſolem FSG, ac præterea tempore, quo planeta vel cometa ſpatium FG abſolvit, invenire latus rectum orbitæ, hincque ipſam orbitam determinare, ſiquidem angulus FSG fuerit minimus.

Solutio.

Sit diſtantia $FS = f$, & $GS = g$, atque angulus $FSG = \phi$ qui cum ponatur minimus, portio ſectionis conicæ FG non ſenſibiliter a linea recta diſcrepabit, erit ergo area FSG proxime triangulum rectilineum, hincque ejus area $= \frac{1}{2} fg \sin \phi$. Ob curvaturam autem arcus FG hæc area erit aliquanto major, ideoque ea propius definietur hac expreſſione $\frac{1}{2} fg \phi$, quippe quæ, ſi orbita fuerit circulus, adeo veram aream præbet. Accuratuis tamen hæc area ita aſſignabitur; ſit $FS = y$, $GS = z$, & angulus $ASF = v$; tum autem ponatur area $ASF = F$ & area $ASG = G$; erit ob $ASG = v + \phi$; area $F = \frac{1}{2} \int yy\,dv$ & $G = \frac{1}{2} \int zz\,dv$. Sit diſtantia perihelii a ſole $AS = a$, & ſemilatus rectum $= b$; erit $y = \frac{ab}{a + (b-a)\cos v}$ & $z = \frac{ab}{a + (b-a)\cos(v+\phi)}$, hinc fit $dy = \frac{ab(b-a)\,dv \sin v}{(a+(b-a)\cos v)^2} = \frac{(b-a)yy\,dv \sin v}{ab}$, ideoque $\frac{dy}{dv} = \frac{(b-a)yy \sin v}{ab}$. Cum nunc ex y naſcatur z, ſi loco v ſcribatur $v+\phi$, erit $z = y + \frac{\phi\,dy}{dv} + \frac{\phi^2 ddy}{2dv^2} +$

$\phi^3 d^3 y$

$\frac{\phi^3 d^3 y}{6 dv^3} +$ &c. posito dv constante. Cum autem simili modo ex area F nascatur area G, si loco v scribatur $v + \phi$ erit G $=$ F $+ \frac{\phi dF}{dv} + \frac{\phi^2 ddF}{2dv^2} + \frac{\phi^3 d^3 F}{6dv^3} +$ &c. ideoque area quæsita FSG $=$ G $-$ F $= \frac{\phi dF}{dv} + \frac{\phi^2 ddF}{2dv^2} + \frac{\phi^3 d^3 F}{6dv^3} +$ &c. & ob F $= \frac{1}{2} \int yy dv$ erit $\frac{dF}{dv} = \frac{1}{2} yy$; hincque porro $\frac{ddF}{dv^2} = \frac{ydy}{dv}$; $\frac{d^3 F}{dv^3} = \frac{yddy + dy^2}{dv^2}$; $\frac{d^4 F}{dv^4} = \frac{yd^3y + 3dyddy}{dv^3}$; $\frac{d^5 F}{dv^5} = \frac{yd^4y + 4dyd^3y + 3ddy^2}{dv^4}$, &c. Hinc ergo prodit G $-$ F $= \frac{1}{2} yy\phi + \frac{\phi^2 ydy}{2dv} + \frac{\phi^3 (yddy + dy^2)}{6dv^2} + \frac{\phi^4 (yd^3y + 3dyddy)}{24dv^3} +$ &c. at est $\frac{1}{2} yz\phi = \frac{1}{2} yy\phi + \frac{\phi\phi ydy}{2dv} + \frac{\phi^3 yddy}{4dv^2} + \frac{\phi^4 yd^3y}{12dv^3} +$ &c. Quo ipsius $\frac{1}{2} yz\phi$ valore subtracto remanebit G $-$ F $= \frac{1}{2} \phi yz + \frac{\phi^3 (2dy^2 - yddy)}{12dv^2} + \frac{\phi^4 (3dyddy - yd^3y)}{24dv^3} + \frac{\phi^5 (6ddy^2 + 8dyd^3y - 3yd^4y)}{240dv^4} +$ &c. Quare cum sit $\frac{dy}{dv} = \frac{(b-a)yy \cos v}{ab}$; erit $\frac{ddy}{dv^2} = \frac{(b-a)yy \cos v}{ab} +$

$$\frac{2(b-a)\,y\,dy\,\sin v}{a\,b\,dv} = \frac{(b-a)\,y\,y\,\cos v}{a\,b} + \frac{2(b-a)^2 y^3 (\sin v)^2}{a^2 b^2};$$

$$\frac{d^3 y}{dv^3} = \frac{-(b-a)\,y\,y\,\sin v}{a\,b} + \frac{6\,(b-a)^2\,y^3\,\sin v\,\cos v}{a^2 b^2} +$$
$$\frac{6(b-a)^3 y^4 (\sin v)^3}{a^3 b^3};\ \&\ \frac{d^4 y}{dv^4} = -\frac{(b-a)\,y\,y\,\cos v}{a\,b} -$$
$$\frac{8(b-a)^2\,y^3\,(\sin v)^2}{a\,a\,b\,b} + \frac{6\,(b-a)^2\,y^3\,(\cos v)^2}{a^2 b^2}, +$$
$$\frac{36(b-a)^3 y^4 (\sin v)^2 \cos v}{a^3 b^3} + \frac{24(b-a)^4 y^4\,(\sin v)^4}{a^4 b^4}.\ \&c.$$ Ex

his conficitur $\frac{2d\,y^2 - y\,d\,d\,y}{dv^2} = -\frac{(b-a)\,y^3\,\cos v}{ab};$

$$\frac{3dyddy - yd^3y}{dv^3} = \frac{(b-a)\,y^3\sin v}{ab} - \frac{3(b-a)^2 y^4 \sin v\,\cos v}{a^2 b^2};$$

$$\frac{6d\,d\,y^2 + 8\,d\,y\,d^3\,y - 3\,y\,d^4 y}{dv^4} = \frac{3(b-a)y^3\cos v}{ab} +$$
$$\frac{4(b-a)^2 y^4 (4(\sin v)^2 - 3(\cos v)^2}{a^2 b^2} - \frac{36\,(b-a)^3\,y^5\,(\sin v)^2\cos v}{a^3 b^3}.$$

Erit ergo area $G - F = \frac{1}{2}\Phi y z - \frac{\Phi^3\,(b-a)\,y^3\cos v}{12ab} +$

$$\frac{\Phi^4 (b-a)\,y^3 \sin v}{24ab^2} - \frac{\Phi^4 (b-a)^2 y^4 \sin v \cos v}{8a^2 b^2} + \frac{\Phi^5 (b-a) y^3 \cos v}{80ab} +$$
$$\frac{\Phi^5\,(b-a)^2 y^4\,(\sin\,v)^2}{15a^2 b^2} - \frac{\Phi^5\,(b-a)^2 y^4\,(\cos v)^2}{20a^2 b^2} -$$
$$\frac{5\Phi^5 (b-a)^3 y^5 (\sin v)^2 \cos v}{20a^3 b^3} + \&c. \text{ atque } z = y + \frac{\Phi (b-a) y^2 \sin v}{ab} +$$

$\Phi^2\,(b-a)$

$\frac{\phi^2 (b-a)\, y\, y \cos v}{2ab} + \frac{\phi^2 (b-a)^2 y^3 \,(\sin v)^2}{a^2 b^2}$. Cum igitur sit $\cos v = \frac{a(b-y)}{y(b-a)}$, & $\cos(v+\phi) = \cos v \cos\phi - \sin v.$ $\sin\phi = \frac{a(b-z)}{z(b-a)}$, erit $\sin v = \frac{a(b-y)}{y(b-a)}.\ \cot\phi - \frac{a(b-z)}{z(b-a)}$ cosec. ϕ. Substituatur tantum valor pro $\cos v$, erit $G - F =$ $\frac{1}{2}\phi\, y\, z - \frac{\phi^3 y y (b-y)}{12 b} + \frac{\phi^4 \,(b-a)\, y^3 \sin v}{24 ab} - \frac{\phi^4 (b-a)\, y^3 (b-y) \sin v}{8abb}$ & loco $\sin v$ valore substituto $G - F = \frac{1}{2}\phi\, y\, z - \frac{\phi^3 yy\,(b-y)}{126} + \frac{\phi^4}{24 bb} y^3 \,(3y - 2b)$ $\left(\frac{b-y}{y}\cot\phi - \frac{(b-z)}{z}\text{cosec.}\,\phi\right)$. Quia autem nulla est ratio, cur y magis in sit quam z, hanc expressionem ita adornabimus, ut vero proxime sit $G - F = \frac{1}{2}\phi\, y\, z - \frac{1}{12}\phi^3$ $y\, z + \frac{\phi^3 y\, z \sqrt{y\, z}}{12 b}$ rejectis terminis sequentibus. Cum vero sit $\sin\phi = \phi - \frac{1}{6}\phi^3 +$ &c. erit hæc area FSG $= \frac{1}{2}$ $y\, z \sin\phi + \frac{\phi^3 y z \sqrt{y\, z}}{12 b}$. Sit jam tempus, quo spatium FG absolvitur $=$ T dierum; erit $T = \frac{y\, z \sin\phi}{2m\sqrt{b}} + \frac{\phi^3\, y\, z \sqrt{y\, z}}{12 m b \sqrt{b}}$ existente $m = 271989, 735$. Quoniam $\sin\phi$ non multum a ϕ discrepat, ponatur $T = \frac{yz \sin\phi}{2m\sqrt{b}} + \frac{y z \sqrt{y\, z}}{12 m b\sqrt{b}} (\sin\phi)^3$,

& for-

& formetur ad b inveniendum hæc æquatio $\frac{\sqrt{yz}}{\sqrt{b}}$ ſin $\phi =$ $\frac{2mT}{\sqrt{yz}} + Q$, fietque $T = T + \frac{Q\sqrt{yz}}{2m} + \frac{2m^2T^3}{3yz\sqrt{yz}}$, hincque $Q = \frac{-4m^3T^3}{3y^2z^2}$. Ex his obtinetur $\frac{\sqrt{yz}}{\sqrt{b}}$ ſin $\phi =$ $\frac{2mT}{\sqrt{yz}} - \frac{4m^3T^3}{3y^2z^2}$, & $\frac{\text{ſin}\,\phi}{\sqrt{b}} = \frac{2mT}{yz} - \frac{4m^3T^3}{3y^2z^2\sqrt{yz}}$. Erit ergo $\frac{\sqrt{b}}{\text{ſin}\,\phi} = \frac{yz}{2mT} + \frac{mT}{3\sqrt{yz}}$, ideoque ſemilatus rectum $b = \left(\frac{yyzz}{4m^2T^2} + \frac{1}{3}\sqrt{yz}\right)(\text{ſin}\,\phi)^2$. Quo cognito orbita per probl. III definietur. Q. E. J.

Coroll. 1.

25. Si ergo cometæ duæ fuerint cognitæ a Sole diſtantiæ $FS = f$, & $GS = g$, una cum angulo $FSG = \phi$ ac præterea obſervatum ſit tempus per ſpatium FG, quod ſit $= T$ dier. erit orbitæ ſemilatus rectum $b = \left(\frac{ffgg}{4m^2T^2} + \frac{1}{3}\sqrt{fg}\right)$ $(\text{ſin}\,\phi)^2$, calculus autem ſæpe facilius inſtituetur quærendo $\sqrt{b} =$ $\left(\frac{fg}{2mT} + \frac{mT}{3\sqrt{fg}}\right)$ ſin ϕ.

Coroll. 2.

26. Hinc ex cognito tempore T area FSG propius definiri, & quantum ſuperet triangulum rectilineum FSG ducta corda FG comprehenſum, aſſignari poteſt. Exceſſus ſcilicet

scilicet erit segmentum FG inter arcum curvæ & cordam contentum, quodest $= \frac{fg\sqrt{fg}}{12b}(\sin \Phi)^3 = \frac{m^2 T^2 \sin \Phi}{3\sqrt{fg}}$

Coroll. 3.

27. Cognito hoc modo latere recto, cujus semissis $= b$, statim reperietur positio perihelii A, posito enim angulo ASF $= v$, erit tang $v = \cot \Phi - \frac{(b-g)f}{(b-f)\sin \Phi}$; hincque porro definitur distantia perihelii a sole AS $= a = \frac{bf\cos v}{b-f+f\cos v}$.

Coroll. 4.

28. Inventis ergo anomalia vera ASF, & rectis a & b assignari poterit tempus, quod ad spatium AF absolvendum impenditur, hincque cum datum sit tempus, quo planeta vel cometa in loco F hæsit, definiri poterit temporis momentum, quo in perihelio est versatus.

Scholion 3.

29. Ad hoc ergo tempus definiendum inserviet methodus in probl. IV. exposita, cujus tres constituendi sunt casus, quorum primus locum habet, si curva AFG fuerit ellipsis seu $2a > b$. Hoc casu quæratur angulus ω, ita ut sit tang $\frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}$ tang $\frac{1}{2}v$; hocque cognito erit tempus per spatium AF $= \frac{a^3}{2m(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\sin \omega\right)$ dierum. Alter casus adhiberi debet, si curva AFG fuerit hyperbola, tum ob

 $b > 2a$

$b > 2a$ quæratur angulus ω, ut ſit tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$. quo cognito erit tempus per AF $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}$ $\left(\frac{b-a}{a}\text{ tang }\omega - l\text{ tang }(45° + \frac{1}{2}\omega)\right)$. Tertius caſus pro curva AFG parabola eſt conſiderandus, ſi ſit $b = 2a$, tum ſumto $t = $ tang $\frac{1}{2}v$, erit tempus per AF $= \frac{a\sqrt{a}}{m\sqrt{2}}(t + \frac{1}{3}t^3)$ dierum. Quemadmodum autem calculus pro parabola eſt facillimus, ita idem maxime impeditur, ſi orbita tantum proxime ad parabolam acceſſerit, Dum enim $2a - b$ vel $b - 2a$ quantitas minima evadit, cum angulus ω ſit nimis parvus, tum vero in temporis expreſſione denominator tam exiguus, ut minimus error in angulo ω commiſſus maximam aberrationem in tempore parere poſſit. Hanc ob rem his caſibus, quibus orbita cometæ proxime ad parabolam accedit, conveniet tempus idonea approximatione exhibitum potius quam verum uſurpare.

Problema VI.

Fig. 3. 30. Si orbita Cometæ non multum a parabola diſcrepet ſive ſit ellipſis ſive hyperbola, definire tempus, quo data quævis. Anomalia vera ASM conficitur.

Solutio.

Sit ut hactenus diſtantia AS $= a$, & ſemi-latus rectum $= b$, quoniam $2a$ & b non multum a ſe invicem diſcrepant, pona-

ponatur $2a - b = \delta$, erit δ quantitas minima, & affirmativa quidem, si curva fuerit ellipsis, at negativa si curva sit hyperbola. Jam sit anomalia vera proposita ASM $= v$, ac ponatur tang $\frac{1}{2} v = t$, erit ut supra (17) vidimus area ASM $= \frac{\int aabbdt(1+tt)}{(b+\delta tt)^2}$; quæ statuatur $= \frac{aabbz}{b+\delta tt}$, atque ob $b + \delta = 2a$ reperietur hic valor pro $z = \frac{t}{b} + \frac{2at^3}{3bb} - \frac{2a\delta t^5}{3.5b^3} + \frac{2a\delta^2 t^7}{5.7b^4} - \frac{2a\delta^3 t^9}{7.9b^5}$ &c. ergo area ASM $=$

$$\frac{aab}{b+\delta tt}\left(t + \frac{2at^3}{3b} - \frac{2a\delta t^5}{3.5b^2} + \frac{2a\delta^2 t^7}{5.7b^2} - \frac{2a\delta^3 t^9}{7.9b^4} + \text{\&c.}\right)$$

Vel cum sit $\frac{b}{b+\delta tt} = 1 - \frac{\delta tt}{b} + \frac{\delta^2 t^4}{bb} - \frac{\delta^3 t^6}{b^3} + $ &c.

$$\text{erit area ASM} = aa\left(t + \tfrac{1}{3}t^3 - \frac{4a\delta}{5bb}t^5 + \frac{6a\delta\delta}{7b^3}t^7 - \frac{8a\delta^3}{9b^4}t^9 \text{ \&c.}\right. \\ \left. + \frac{\delta\delta}{bb}t^5 - \frac{\delta^3}{b^3}t^7 + \frac{\delta^4}{b^4}t^9\right]$$

Quare cum tempus sit $= \frac{\text{ar. ASM}}{m\sqrt{b}}$ dierum, erit tempus quo anomalia vera ASM $= v$ absolvitur in diebus expressum

$$= \frac{aa}{m\sqrt{b}}\left(t + \tfrac{1}{3}t^3 - \frac{4a\delta}{5bb}t^5 + \frac{6a\delta^2}{7b^3}t^7 - \frac{8a\delta^3}{9b^4}t^9 \text{ \&c.}\right. \\ \left. + \frac{\delta\delta}{bb}t^5 - \frac{\delta^3}{b^3}t^7 + \frac{\delta^4}{b^4}t^9\right]$$

quæ expressio, etsi in infinitum progreditur, tamen citissime

convergit, si $\delta = 2a - b$ fuerit quantitas vehementer parva, uti statuimus. Q. E. J.

Coroll. 1.

31. Quia est $2a = b + \delta$, si in serie infinita hic valor ubique loco $2a$ substituatur, reperietur tempus, quo anomalia vera ASM $= v$, cujus semissis tangens ponitur $= t$, absolvitur

$$= \frac{aa}{m\sqrt{b}}\left(\begin{array}{l} t + \frac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \\ \qquad\qquad + \frac{3\delta\delta}{5bb}t^5 - \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9 \end{array} \&c.\right]$$

Coroll. 2.

32. Quoniam pro hyperbola fit δ numerus negativus, omnes termini prodibunt affirmativi, si enim pro hyperbola ponatur $b - 2a = \delta$ erit tempus per arcum AM $= \frac{aa}{m\sqrt{b}}$

$$\left(\begin{array}{l} t + \frac{1}{3}t^3 + \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 + \frac{4\delta^3}{9b^3}t^9 \\ \qquad\qquad + \frac{3\delta\delta}{5bb}t^5 + \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9 \end{array} \&c.\right]$$

Coroll. 3.

33. Series hæ maxime convergunt, quo minor fuerit angulus ASM; sin autem hic angulus v fiat tantus, ut ejus semissis multum superet semirectum, ideoque ejus tangens t unitatem longe superet, tum convergentia diminuetur. His ergo casibus expediet methodo directa uti. Quando quidem angulus ille ω ita sumtus, ut sit tang $\frac{1}{2}\omega = \frac{\sqrt{(2a - b)}}{\sqrt{b}}$

tang

tang $\frac{1}{2}v$, vel tang $\frac{1}{2}\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}v$, prodit adhuc notabilis magnitudinis.

Problema VII.

34. Si orbita cometæ non admodum a parabola discrepet, ex dato tempore, quod vel ante vel post appulsum ad perihelium elapsum sit, locum cometæ in Orbita, hoc est anomaliam veram ASM, una cum distantia cometæ a sole invenire. Fig. 3

Solutio.

Quoniam primo orbita cometæ est data, ponatur distantia perihelii a sole A S $= a$, semilatus rectum $= b$; & quoniam orbita non multum a parabola differre statuitur posito $2a - b = \delta$, erit δ quantitas valde parva. Deinde sit tempus, quod vel ante appulsum ad perihelium effluxerit, vel post, in diebus expressum $=$ T; Anomalia vera autem, quæ quæritur, ASM sit $= v$, ac $t =$ tang $\frac{1}{2}v$. Jam ex præcedente problemate habebimus hanc æquationem

$$T = \frac{aa}{m\sqrt{b}}\left(t + \frac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \quad \&c.\right.$$
$$\left. + \frac{3\delta\delta}{5bb}t^5 - \frac{4\delta^3}{7b^3}t^7 + \frac{5\delta^4}{9b^4}t^9 \right]$$

ex qua valorem ipsius t erui oportebit. Ponamus $\frac{mT\sqrt{b}}{aa} = n$ brevitatis gratia, ut sit $n = t + \frac{1}{3}t^3$

$$\begin{array}{l} -\frac{2\delta}{5b}\,t^5 + \frac{3\delta\delta}{7bb}\,t^7 - \frac{4\delta^3}{9b^3}\,t^9 \\ +\frac{3\delta\delta}{5bb} \qquad -\frac{4\delta^3}{7b^3} \qquad +\frac{5\delta^4}{9b^4} \end{array}$$

&c. Sit primum orbita cometæ vera parabola, ideoque $\delta = o$, erit $n = t + \frac{1}{3}t^3$, atque $t^3 + 3t - 3n = o$; cujus æquationis cubicæ per regulam Cardani radix erit:

$$t = \sqrt[3]{\left(\tfrac{3}{2}n + V\left(\tfrac{9}{4}nn + 1\right)\right)} - \sqrt[3]{\left(-\tfrac{3}{2} + V\left(\tfrac{9}{4}nn + 1\right)\right)} \text{ vel}$$

$$t = \sqrt[3]{\left(\tfrac{3}{2}n + V\left(\tfrac{9}{4}nn + 1\right)\right)} - \frac{1}{\sqrt[3]{\left(\tfrac{3}{2}n + V\left(\tfrac{9}{4}nn + 1\right)\right)}} \text{ vel etiam}$$

$$t = \frac{1}{\sqrt[3]{\left(-\tfrac{3}{2}n + V\left(\tfrac{9}{4}nn + 1\right)\right)}} - \sqrt[3]{\left(-\tfrac{3}{2}n + V\left(\tfrac{9}{4}nn + 1\right)\right)}$$

Vel igitur ex his formulis, vel ex tabulis in hunc finem computatis valor ipsius t erui poterit, quo cognito dabitur anomalia vera ASM $= v$, per æquationem tang $\frac{1}{2}v = t$. Deinde autem habebitur distantia cometæ a sole SM $= y = \frac{ab}{a + (b-a)\cos v}$. Cum vero pro parabola sit $b = 2a$; erit $y = \frac{2a}{1 + \cos v} = \frac{a}{(\cos \frac{1}{2}v)^2}$. Sic itaque si orbita cometæ vera fuerit parabola, determinabitur ad datum tempus locus cometæ in orbita.

Sin autem orbita cometæ fuerit ellipsis maxime excentrica vel hyperbola non multum a parabola abhorrens, ita ut sit $\delta = 2a - b$ quantitas vehementer parva, posito $\frac{mTVb}{aa} = n$, hæc æquatio resolvi debebit:

$n = t$

$$n = t + \tfrac{1}{3}t^3 - \frac{2\delta}{5b}t^5 + \frac{3\delta\delta}{7bb}t^7 - \frac{4\delta^3}{9b^3}t^9 \ \&c.$$
$$+ \frac{3\delta\delta}{5bb} \quad - \frac{4\delta^3}{7b^3} \quad + \frac{5\delta^4}{9b^4}$$

Sumatur primo tantum hæc æquatio $n = t + \frac{1}{3}t^3$, & per modum præcedentem quæratur valor ipsius t, sitque $t = \theta$, ita ut sit $n = \theta + \frac{1}{3}\theta^3$; eritque ob δ valde parvam quantitatem θ valor ipsius t vero proximus. Sit igitur verus valor $t = \theta + A\theta^3 + B\theta^5 + C\theta^7 + \&c.$ erit $t^3 = \theta^3 + 3A\theta^5 + 3B\theta^7$ &c.
$$+ 3A^2\theta^7$$
$$t^5 = \theta^5 + 6A\theta^7, \&c. \quad \& \ t^7 = \theta^7.$$
quibus substitutis in æquatione orietur $n = \theta + \frac{1}{3}\theta^3 =$
$$\theta + A\theta^3 + B\theta^5 + C\theta^7$$
$$+ \tfrac{1}{3}\theta^3 + A\theta^5 + B\theta^7 + \&c.$$
$$- \frac{2\delta}{5b}\theta^5 + A^2\theta^7$$
$$+ \frac{3\delta\delta}{5bb}\theta^5 - \frac{2\delta}{b}A\theta^7 \ \&c.$$
$$+ \frac{3\delta\delta}{bb}A\theta^7$$
$$+ \frac{3\delta^3}{7bb}\theta^7$$
$$- \frac{4\delta^3}{7b^3}\theta^7$$

Singulis ergo terminis more solito ad nihilum reductis erit

$A = 0;$

$$A = o; \quad B = \frac{2\delta}{5b} - \frac{3\delta\delta}{5bb}; \quad C = -B - \frac{3\delta\delta}{7bb} + \frac{4\delta^3}{7b^3} =$$

$$-\frac{2\delta}{5b} + \frac{6\delta\delta}{35bb} + \frac{4\delta^3}{7b^3} = \frac{2\delta}{b}\left(\frac{\delta}{b} + 1\right)\left(\frac{2\delta}{7b} - \frac{1}{5}\right)$$

Cognito ergo ipsius t valore vero proximo θ, erit verus valor

$$t = \theta + \left(\frac{2\delta}{5b} - \frac{3\delta\delta}{5bb}\right)\vartheta^5 - \left(\frac{2\delta}{5b} - \frac{6\delta\delta}{35bb} - \frac{4\delta^3}{7b^3}\right)\vartheta^7:$$

Tum vero ob $t =$ tang $\frac{1}{2}v$ cognoscetur anomalia vera $ASM = v$; hincque porro prodibit distantia cometæ a sole

$$SM = y = \frac{ab}{a + (b-a)\cos v} = \frac{b}{1 + \frac{b-a}{a}\cos v}. \quad \text{Q. E. J.}$$

Coroll. 1.

35. Pro ellipsi ergo maxime oblonga δ ob quantitatem affirmativam, erit verus valor ipsius t major quam valor θ, qui ex hypothesi orbitæ parabolicæ est erutus.

Coroll. 2.

36. Pro hyperbola autem ubi fit δ quantitas negativa verus valor ipsius t minor erit quam θ; utroque autem casu ob tantas ipsius θ potestates, expressio pro t vehementer convergit, siquidem sit $\theta < 1$, quod fit si anomalia vera v fuerit angulo recto minor.

Scholium.

37. Quod si autem anomalia vera v multum excedat angulum rectum, ita ut δ vel t fiat numerus unitate multo major, hincque expressio illa pro t inventa divergat potius quam convergat, tum ista methodo uti non conveniet, nisi forte δ sit

quan-

quantitas tantopere exigua, ut per eam omnes termini efficiantur minimi. Oportebit igitur his casibus expressionem veram, qua relatio inter tempus & anomaliam veram exhibetur, in subsidium vocare; atque ex ea modum, quo ad datum tempus anomalia vera assignari possit, eruere. Hanc itaque operationem in sequentibus problematis fusius sum ostensurus, ut quovis casu oblato theoria adhiberi, per eamque verus motus tam cometarum quam planetarum definiri queat.

Problema VIII.

38. Si orbita planetæ vel cometæ fuerit ellipsis quæcunque cognita, ad datum tempus, vel ante vel post appulsum ejus ad perihelium A, ejus anomaliam veram ASM, atque distantiam a sole SM assignare.

Solutio.

Sit perihelii a sole distantia AS $= a$, & semilatus rectum $= b$; erit $2a > b$, quia orbita ponitur ellipsis; Tempus autem, quod vel ante vel post appulsum ad perihelium effluxerit, sit in diebus expressum $=$ T. Nunc ponatur anomalia vera, quæ quæritur ASM $= v$, sitque alius angulus ω ita comparatus ut sit $\operatorname{tang} \frac{1}{2}\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}} \operatorname{tang} \frac{1}{2} v$, eritque ut vidimus (19), tempus $T = \frac{a^3}{2m(2a-b)^{\frac{3}{2}}} \left(\omega - \frac{(b-a)}{a} \sin\omega\right)$ ex qua æquatione angulum ω erui oportet. Fiet ergo $\omega - \frac{(b-a)}{a}$ $\sin\omega =$

$\sin\omega = \frac{2m(2a-b)^{\frac{3}{2}}T}{a^3}$. Convertatur hæc expreſſio $\frac{2m(2a-b)^{\frac{3}{2}}T}{a^3}$ quæ eſt cognita, in arcum circuli cujus radius $= 1$, quod ita fiet: Sumatur expreſſionis $\frac{2m(2a-b)^{\frac{3}{2}}T}{a^3}$ logarithmus, ad quem addatur conſtanter hic logarithmum 5, 3144251332; atque ſumma erit logarithmus anguli hujus quæſiti in minutis ſecundis expreſſi. Vel cum ſit logarithmus $m = 5,4345525139$, ad log. $\frac{(2a-b)^{\frac{3}{2}}T}{a^3}$ addatur conſtanter iſte logarithmus 11, 0500076428, & numerus logarithmo reſultanti reſpondens dabit angulum in minutis ſecundis expresſum. Sit iſte angulus $= u$, qui erit ille ipſe, quem Aſtronomi anomaliam mediam vocare ſolent, qui ergo hoc pacto ex dato tempore T inveniri, vel ex tabulis aſtronomicis, ſi quæſtio de planeta quopiam inſtituatur, depromi poteſt. Priori autem modo ſi iſte angulus u in minutis ſecundis exprimatur erit $lu = 11,0500076428 + \frac{3}{2}l(2a-b) + lT - 3la$, unde deficientibus tabulis aſtronomicis, anomalia media u expedite reperitur. Inventa erga jam ſit hæc anomalia media u, eritque $u = \omega - \frac{(b-a)}{a}\sin\omega$. Hæc æquatio commodiſſime aliquot tentaminibus reſolvetur. Scilicet cum ſit $\omega > u$, ſumatur pro arbitrio angulus pro ω, & computetur angulus $= \frac{(b-a)}{a}$ $\sin\omega$,

$\sin\omega$, quod fiet, si a $l\,\frac{b-a}{a} + l\sin\omega$ ex tabulis sumto subtrahatur 4, 6855748668, logarithmus enim residuus dabit numerum minutorum secundorum ipsi $\frac{(b-a)}{a}\sin\omega$ æquivalentium; hoc igitur facto pro $\omega - \frac{(b-a)}{a}\sin\omega$ reperietur angulus vel major vel minor quam u; priori casu valor pro ω assumtus erat nimis magnus, posteriori vero nimis parvus; sic igitur corrigendo hypothesin factam mox angulus ω prope verus cognoscetur. Sit valor jam prope verus pro ω inventus $=\varrho$; verus autem valor sit $\omega = \varrho + z$, erit $\sin(\varrho + z) = \sin\varrho + z\cos\varrho$; ideoque $u = \varrho + z - \frac{(b-a)}{a}\sin\varrho - (b-u)\,z\cos\varrho$ & hinc nascitur $z = \dfrac{u - \varrho + \frac{(b-a)}{a}\sin\varrho}{1 - \frac{(b-a)}{a}\cos\varrho}$, ubi in numeratore pars $\frac{b-a}{a}\sin\varrho$ pari modo in angulum converti debet, ut supra ostendimus. Denominator autem erit merus numerus, & ob sinum totum $=1$, logarithmi $\cos\varrho$ characteristica denario minui debet. Hoc ergo modo reperitur angulus z, qui ad angulum ϱ additus dabit verum angulum quæsitum ω. Sin autem angulus ϱ assumtus nimium a vero discesserit, hoc modo multo propior valor pro ω orietur, qui tum loco ϱ substitutus vero proximum valorem pro ω suppeditabit. Quodsi au-

tem hac methodo repertus fuerit angulus ω, hinc facile invenietur anomalia vera ASM $= v$ ope hujus formulæ tang $\frac{1}{2} v$

$$= \frac{\sqrt{b}}{\sqrt{(2a-b)}} \text{ tang } \tfrac{1}{2}\omega.$$ Tandem vero erit diſtantia planetæ ſeu cometæ a ſole SM $= \frac{ab}{a+(b-a)\operatorname{coſ} v}$. Q. E. J.

Coroll. 1.

39. Coefficiens $\frac{b-a}{a}$, qui in hoc calculo occurrit, vocari ſolet orbitæ excentricitas; (9) hujus ergo logarithmus imprimis notari debet, quippe quo cognito totus calculus facillime inſtituetur.

Coroll. 2.

40. Ex data ergo orbitæ excentricitate $\frac{b-a}{a}$ hujus calculi ope ad anomaliam mediam quamvis propoſitam expedite anomalia vera reſpondens definietur: Hocque modo tabulæ æquationis motuum planetarum & cometarum, qui quidem in ellipſibus revolvuntur, ſupputabuntur.

Problema IX.

Fig. 3. 41. Si cometa in hyperbola circa ſolem moveatur, atque tempus, quo per perihelium A tranſit, fuerit notum, ad datum, quodvis tempus locum cometæ in orbita ſeu anomaliam veram ASM, ejusque a ſole diſtantiam SM ſupputare.

Solutio.

Solutio.

Sit iterum perihelii a ſole diſtantia AS $= a$, & ſemilatus rectum $= b$, eritque $b \succ 2a$. Tempus autem propoſitum a momento, quo Cometa in perihelio hæret, diſtet intervallo $=$ T dierum. Deinde ſit anomalia vera, quam inveniri oportet, ASM $= v$; ac concipiatur alius angulus ω, ut ſit tang $\frac{1}{2}\,\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}$ tang $\frac{1}{2}\,v$; eritque (20), T $= \frac{a^3}{2m(b-2a)^{\frac{3}{2}}}\left(\frac{b-a}{a}\text{ tang }\omega - l\text{ tang }(45^\circ + \tfrac{1}{2}\,\omega)\right)$. Statuatur $\frac{2m(b-2a)^{\frac{3}{2}}\,\mathrm{T}}{a^3} = u$, erit u quantitas cognita, atque illi, quæ ante anomaliæ mediæ nomine occurrerat, analoga: hoc vero caſu expedíet u in meris numeris expreſſam retinere, quam in angulum convertere. His monitis erit itaque $u = \frac{b-a}{a}$ tang $\omega - l$ tang $(45^\circ + \frac{1}{2}\,\omega)$ unde tentando pro ω ejusmodi valor eruatur qui a vero non multum diſſideat. Sit ergo ϱ angulus non multum a vero anguli ω valore diſcrepans, ac ponatur $\omega = \varrho + z$, ob z angulum valde parvum erit tang. $\omega = \frac{\text{tang }\varrho + z}{1 - z\text{ tang }\varrho}$ ſeu tang $\omega =$ tang $\varrho + z\,(1 + (\text{tang }\varrho)^2)$ $=$ tang $\varrho + \frac{z}{(\cos \varrho)^2}$. Ad l tang $(45^\circ + \frac{1}{2}\,\omega)$ inveniendum ponatur l tang $(45^\circ + \frac{1}{2}\varrho) = \mathrm{R}$, eritque l tang $(45^\circ + \frac{1}{2}\,\omega)$ valor ipſius R, qui prodit ſi loco ϱ ſcribatur $\varrho + z$,

 prodit

prodit autem is $= R + \frac{z\, d\, R}{d\varrho}$. Erit vero $d\, R =$ $\frac{\frac{1}{2} d\varrho}{(\cos(45^\circ + \frac{1}{2}\varrho))^2 \operatorname{tang}(45^\circ + \frac{1}{2}\varrho)} = \frac{d\varrho}{\cos\varrho}$; ideoque l tang $(45^\circ + \frac{1}{2}\omega) = l \operatorname{tang}(45^\circ + \frac{1}{2}\varrho) + \frac{z}{\cos\varrho}$. His substitutis erit

$$u = \frac{b-a}{a} \operatorname{tang} \varrho + \frac{(b-a)z}{a(\cos\varrho)^2} - l \operatorname{tang}(45 + \tfrac{1}{2}\varrho) - \frac{z}{\cos\varrho},$$

unde nascitur $z = \dfrac{u + l \operatorname{tang}(45^\circ + \frac{1}{2}\varrho) - \frac{(b-a)}{a} \operatorname{tang} \varrho}{\frac{1}{(\cos\varrho)^2}\left(\frac{b-a}{a} - \cos\rho\right)}$

seu $z = \dfrac{u + l \operatorname{tang}(45 + \frac{1}{2}\varrho) - \frac{(b-a)}{a} \operatorname{tang}\varrho}{\frac{(b-a)}{a} - \cos\rho}(\cos\varrho)^2$. Hoc modo reperietur numerus pro z, dummodo ea regula ad l tang $(45 + \frac{1}{2}\rho)$ exprimendum observetur, quæ supra (22) est tradita. Hic autem numerus pro z inventus in angulum est convertendus, modo, quem ante exposui. Hisque factis habebitur verus valor anguli $\omega = \varrho + z$, qui autem, si adhuc dubium supersit, denuo pro ρ ipse poni, sicque ulterius corrigi poterit. Invento autem angulo ω, statim reperietur tang $\frac{1}{2} v = \frac{\sqrt{b}}{\sqrt{(b - 2a)}}$ tang $\frac{1}{2}\omega$, atque adeo definietur anomalia vera quæsita ASM $= v$, ex qua porro elicitur distantia cometæ a sole SM $= y =$ $\frac{ab}{a + (b-a)\cos v}$. Q. E. J.

Scholion.

Scholion.

42. Ad usum hujus calculi, qui minus est usitatus in astonomia, uberius declarandum, ponamus cometam in hyperbola æquilatera deferri, atque perihelii ejus a sole distantiam esse = distantiæ terræ a sole mediæ, nempe esse . . . . $a = 100000$
unde fiet $b = a(1 + \sqrt{2})$, seu $b = 241421,356$
& quæri debeat locus hujus cometæ centum diebus post transitum ejus per perihelium; ita ut sit $T = 100$. Primo ergo quæri debet u ut sit $l\,u = l\,2m + l\,T + \frac{3}{2}\,l\,41421,356 - 3\,l\,100000$; unde sequens calculus nascitur,

$l\,2m$	=	5, 7 3 5 5 8 2 5
$l\,T$	=	2, 0 0 0 0 0 0 0
$l\,41421,356$	=	4, 6 1 7 2 2 4 3
semiss.	=	2, 3 0 8 6 1 2 2
		1 4, 6 6 1 4 1 9 0
$3\,l\,100000$	=	1 5, 0 0 0 0 0 0 0
$l\,u$	=	9, 6 6 1 4 1 9 0
unde u	=	0, 4 5 8 5 8 4

Deinde est $\frac{b-a}{a} = 1,41421356 = \sqrt{2}$

ideoque $l\,\frac{b-a}{a} = 0,1505150$

Statuatur primum $\text{tang}\,\omega = \frac{a}{b-a}\,u$; neglecto altero termino

erit ob $l\,u$	=	9, 6 6 1 4 1 9 0
subtr. $l\,\frac{b-a}{a}$	=	0, 1 5 0 5 1 5 0
$l\,\text{tang}\,\omega$	=	9, 5 1 0 9 0 4 0

ideoque

$$\begin{array}{rcl} \text{ideoque } \omega & = & 17^\circ, 58', \\ \text{\& hinc fiet angulus } 45^\circ + \tfrac{1}{2}\omega & = & 53^\circ, 59', \\ \text{cujus tangens eſt} & = & 1,3755403. \end{array}$$

Patet ergo angulum $17^\circ, 58'$ vehementer nimis eſſe parvum & ſi pro ω ſumatur 30°, adhuc iſte valor nimis erit parvus, prodit enim pro u hic numerus 0, 26719, ſin autem ſtatuatur $\omega = 40^\circ$, prodit pro u iſte numerus 0, 4237, quare cum ſit $u =$ 0, 458584, perſpicuum eſt hunc valorem non multum a veritate abludere, atque ex his duabus poſitionibus 30° & 40° conjecturam faciendo reperietur ω propemodum eſſe debere 42°, qui valor ergo pro ϱ accipiatur, ita ut ſit $\varrho = 42^\circ$, & $\tfrac{1}{2}\varrho = 21^\circ$ atque $45^\circ + \tfrac{1}{2}\varrho = 66^\circ$. Calculus itaque ſic inſtituatur

$$\begin{array}{rcl} l\,\mathrm{tang}\, 66^\circ - 10 & = & 0,3514169 \\ \text{hujus log.} & = & 9,5458226 \\ \text{addatur per (22)} & \ldots & 0,3622157 \\ \text{Erit ergo} & & 9,9080383 \\ l\,\mathrm{tang}(45^\circ + \tfrac{1}{2}\varrho) & = & 0,809167 \\ \text{Porro eſt } l\,\mathrm{tang}\,\varrho & = & 9,9544374 \\ \text{add. } l\,\dfrac{b-a}{a} & = & 0,1505150 \\ l\,\dfrac{b-a}{a}\mathrm{tang}\,\varrho & = & 1,1049524 \\ \text{Ergo } \dfrac{b-a}{a}\mathrm{tang}\,\varrho & = & 1,273364 \end{array}$$

$$\begin{array}{rcl} \text{Quo circa ad } u & = & 0,458584 \\ \text{addatur } l\,\mathrm{tang}(45^\circ + \tfrac{1}{2}\varrho) & = & 0,809167 \\ & & 1,267751 \\ \text{ſubtrahatur } \dfrac{b-a}{a}\,\mathrm{tang}\,\varrho & = & 1,273364 \\ \text{erit numerator} & = & -0,005613 \end{array}$$

$a=b$

$\frac{b-a}{a}$	$=$	1,4142135
cos ϱ	$=$	7431448
Denom.	$=$	0,6710687
Jam est l cos ϱ	$=$	9,8710735
dupl. $l\,(\cos\varrho)^2$		9,7421470
Add. l num.		7,7491950
		7,4913420
subtr. l den.	$=$	9,8267670
$l-z$	$=$	7,6645750
subtr.		4,6855749
		2,9790001

ergo $-z = 952,\ 79'' = 15',\ 53''$

Erit ergo $\omega = \rho + z = 41^\circ,\ 44',\ 7''$.

Invento ergo angulo $\omega = 41^\circ,\ 44',\ 7''$, erit tang $\frac{1}{2} v = \frac{\sqrt{b}}{\sqrt{(b-2a)}}$ tang $\frac{1}{2}\omega$: at ob $b = a(1+\sqrt{2})$ erit $\frac{b}{b-2a} = \frac{1+\sqrt{2}}{\sqrt{2}-1} = (1+\sqrt{2})^2$ ideoque $\sqrt{\frac{b}{b-2a}} = 1 + \sqrt{2} = 2,41421356$, & ob $\frac{1}{2}\omega = 20^\circ, 52', 3\frac{1}{2}''$ calculus ita se habebit.

Ad. $l\sqrt{\frac{b}{b-a}}$	$=$	0,3827756
add. l tang $\frac{1}{2}\omega$	$=$	9,5811709
l tang $\frac{1}{2}v$	$=$	9,9639465
Ergo $\frac{1}{2}v$	$=$	$42^\circ,\ 37',\ 28''$
Ac propterea v	$=$	$85^\circ,\ 14',\ 56''$

Atque ideo cometa iste tempore 100 dierum post transitum per perihelium descripsit angulum ASM $= v = 85^\circ$,

14′, 56″. Erit ergo hoc tempore ejus a sole distantia SM $=$

$$y = \frac{ab}{a + (b-a)\cos v} = \frac{b}{1 + \frac{b-a}{a}\cos v}.$$

Quare ad $l \cos v$	$=$	8, 9 1 8 1 7 4 7
add. $l\,\frac{b-a}{a}$	$=$	0, 1 5 0 5 1 5 0
		9, 0 6 8 6 8 9 7
ergo $\frac{b-a}{a}\cos v$	$=$	0, 1 1 7 1 3 6
& denom.	$=$	1, 1 1 7 1 3 6
Jam a $l\,b$	$=$	5, 3 8 2 7 7 5 6
subtr. l denom.	$=$	0, 0 4 8 1 0 6 1
erit $l\,y$	$=$	5, 3 3 4 6 6 9 5
& SM $= y$	$=$	2 1 6 1 0 7.

Cum igitur distantia a Sole in perihelio esset 100000, erit post 100 dies cometæ a solo distantia $=$ 216107. Hocque exemplum ad calculum hujus methodi illustrandum sufficiet.

Problema X.

Fig. 4. 43. Datis duobus planetæ cometæve locis F & H a se invicem non multum remotis una cum tempore, quo spatium FH est confectum, ad quodvis tempus medium ejus locum verum G assignare.

Solutio.

Quo hoc problema commodius resolvere queamus, applicemus id ad motum terræ, quam motu medio in distantia a sole mediocri circulum percurrere concipiamus. Existente ergo sole in s sint f & h duo terræ loca, & tempus per fh sit $=$ T dierum.

dierum. Sit distantia media terræ a sole $fs = hs = c = 100000$ ut hactenus sumsimus, hincque reperietur angulus fsh, ad $l\,\frac{2mT}{c\sqrt{c}}$ addendo 5, 3144251332, numerus enim respondens summæ exhibebit angulum fsh in minutis secundis, seu ob m & c in numeris data, ad l T addatur hic logarithmus 2, 5500076427, & numerus summæ respondens dabit angulum fsh in minutis secundis expressum. Dispertiatur jam tempus T in duas partes α, & β ita ut sit $T = \alpha + \beta$ atque elapsa temporis parte α manifestum est terram futuram esse in g, ita ut ducta gs, sit ang. $fsg : gsh = \alpha : \beta$; erit ergo $fsg = \frac{\alpha}{T}\,fsh$, & $gsh = \frac{\beta}{T}\,fsh$. Ducatur corda fh, secans radium sg in o, & quæratur sagitta og hoc modo: ob ang. $sfo = 90° - \frac{1}{2}fsh$, & $sof = 90 + \frac{1}{2}fsh - \frac{\alpha}{T}\,fsh = 90 + \frac{(\beta-\alpha)}{2T}\,fsh$, erit $so : sf = \operatorname{cos}\frac{1}{2}fsh : \operatorname{cos}\frac{\beta-\alpha}{2T}\,fsh$, hincque $so = c\,\frac{\operatorname{cos}\frac{1}{2}fsh}{\operatorname{cos}\frac{\beta-\alpha}{2T}fsh}$; ac propterea sagitta $go =$

$$\frac{c\left(\operatorname{cos}\frac{\beta-\alpha}{2T}\,fsh - \operatorname{cos}\frac{1}{2}\,fsh\right)}{\operatorname{cos}\frac{\beta-\alpha}{2T}\,fsh} = \frac{2c\operatorname{sin}\frac{\beta}{2T}\,fsh.\ \operatorname{sin}\frac{\alpha}{2T}\,fsh}{\operatorname{cos}\frac{\beta-\alpha}{2T}\,fsh}$$

Est vero $\frac{1}{2T}\,fsh$ angulus constans, qui erit $= 29', 34$

$\frac{98''}{1000}$ seu $= 1774, 098''$, cujus numeri logarithmus est $3, 2489776471$. Quodsi ergo hic angulus $\frac{1}{2T} f s b = 29', 34 \frac{''98}{1000}$ ponatur $= \tau$, & tempora α & β in diebus exprimantur, erit sagitta $g\, o = \frac{2c \sin \beta \tau . \sin \alpha \tau}{\cos (\beta - \alpha) \tau}$.

His præmissis observatus sit planeta seu cometa in F tempore autem $T = \alpha + \beta$ elapso in H, & quæratur ubi is sit futurus elapso tempore tantum α, postquam in F est versatus. Sit G locus quæsitus, & quoniam tempora in eadem orbita sunt ut areæ circa solem S descriptæ, erit area FSG : ar. GSH $= \alpha : \beta$. Ducatur corda FH, secans SG in O, & sequenti modo sagitta GO definietur. Quoniam curvatura orbitæ FGH proficiscitur a vi centripeta, ponamus vim centripetam qua cometa seu planeta spatium FGH peragrans ad solem S sollicitatum, esse constantem $= P$; quia enim loca F & H satis sibi sunt vicina, hæc hypothesis a veritate sensibiliter non recedit; Quo minus autem discrepet, ponamus P eam esse vim centripetam, quæ in loco medio G exercetur, ejusque directionem rectæ SG esse parallelam. Simili modo sit vis centripeta terram in orbita circulari retinens $= p$, quæ quidem per se erit constans, at ponamus pariter, terram ab ea dum spatium $f\,g\,b$ percurrit, constanter in directione $g\,s$ sollicitari. In F & f ducantur tangentes FMN, $f\,m\,n$, & ex H & b ipsis SG & $s\,g$ parallelæ HN & $b\,n$, erunt hæc intervalla HN & $b\,n$ effectus

effectus ab istis viribus centripetis producti: & quia hi effectus eodem tempore $= \alpha + \beta$ producuntur, erunt ii ipsis viribus proportionales, quare erit $HN : hn = P : p$. Erunt porro ex natura motus æquabilis quo tangentes FN & fn describerentur, si nulla adesset vis sollicitans, spatia $FM : MN = \alpha : \beta$; & $fm : mn = \alpha : \beta$, unde ob triangula FOM & FHN similia erit quoque $FO : OH = \alpha : \beta$. Cum igitur & corda FH & ratio $\alpha : \beta$ detur ad locum medium G inveniendum secetur corda FH in O, ut sit $FO : OH = \alpha : \beta$, & radius ex S per O productus transibit per locum G quæsitum. Superest ergo ut sagitta GO definiatur, quod ex effectu virium ita præstabitur: Erit nempe $HN : GM = FN^2 : FM^2 = (\alpha + \beta)^2 : \alpha^2$; & pari modo $hn : gm = fn^2 : fm^2 = (\alpha + \beta)^2 : \alpha^2$, unde fit $HN : GM = hn : gm$ seu $HN : hn = GM : gm$: est vero quoque $HN : hn = OM : om$; unde concluditur $HN : hn = OG : og = P : p$; ideoque erit $OG = \frac{P}{p}\, og$. Cum igitur sit $og = \frac{2c \sin \alpha\tau, \sin \beta\tau}{\cos(\beta - \alpha)\tau}$; erit sagitta quæsita $OG = \frac{2Pc \sin \alpha\tau . \sin \beta\tau}{p \cos(\beta - \alpha)\tau}$. Divisa ergo corda FH in ratione temporum $\alpha : \beta$ in O, ductaque SO si capiatur $OG = \frac{2Pc \sin \alpha\tau, \sin \beta\tau}{p \cos(\beta - \alpha)\tau}$ erit G locus planetæ seu cometæ quæsitus, siquidem loca extrema F & H non multum sint a se invicem remota. Cum autem vires centripetæ sint in ratione reciproca duplicata distantiarum a sole, erit

 $P, p =$

$P:p = cc;\ SG^2$: vel potius, quia vim sumi oportet mediam inter extremas, erit ad veritatem accommodatius $P:p = 4cc:(SF + SH)^2$. Hac virium centripetarum ratione in computum ducta erit sagitta $OG = \frac{8c^3 \sin \alpha\tau . \sin \beta\tau}{(SF+SH)^2 \cos(\beta-\alpha)\tau}$. Q. E. J.

Corol. 1.

44. Si ratio temporum $\alpha:\beta$, atque adeo ratio segmentorum cordæ $FO:OH$ non multum a ratione æqualitatis discrepet, tum sine errore loco $\frac{FS+SH}{2}$ scribi poterit SG ita ut sit $OG = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 \cos(\beta-\alpha)\tau}$. Ac si ratio $\alpha:\beta$ fuerit penitus ratio æqualitatis erit $OG = \frac{2c^3 \sin(\alpha\tau)^2}{SG^2} = \frac{c^3 \int v{:}2\alpha\tau}{SG^2}$.

Coroll. 2.

45. Si ergo vicissim detur locus G, in rectis SF & SH, quæ repræsentant loca heliocentrica, puncta F & H assignari poterunt, in quibus planeta seu cometa tempore α dier. antequam ad G, & tempore β dier. postquam in G fuerat, versatur. Nam ex puncto G, sumatur $GO = \frac{2c^3 \sin\alpha\tau . \sin\beta\tau}{SG^2 . \cos(\beta-\alpha)\tau}$ & per punctum O agatur recta FOH, ita ut sit $FO:HO = \alpha:\beta$, erunt F & H loca quæsita.

Coroll. 3.

47. Quo minora ergo fuerint temporum intervalla α & β & quo propius ad rationem æqualitatis accedant, eo minus

nus hæc determinatio a veritate aberrabit. Neque vero plerumque aberratio a veritate fit ſenſibilis, niſi angulus FSH ſit ſatis magnus & circiter 10 vel 15 gradus ſuperet, quæ latitudo ſatis eſt magna, ut hinc utiles methodi ad orbitas tam planetarum quam cometarum deriventur.

Problema XI.

47. Datis planetæ ſeu cometæ tribus locis geocentricis a ſe invicem non nimis remotis, una cum ejus diſtantia vera a terra tempore obſervationis mediæ; definire ejus planetæ ſeu cometæ orbitam veram, in qua circa ſolem movetur.

Solutio.

Tempore primæ obſervationis ſit terra in f exiſtente ſole in S, ſitque longitudo ſolis $= f$. In obſervatione ſecunda ſit terra in g, & longitudo ſolis $= g$; in obſervatione tertia ſit terra in H, & longitudo ſolis $= h$. Tempus autem inter primam & ſecundam obſervationem ſit $= \alpha$; & tempus inter ſecundam & tertiam obſervationem $= \beta$. Erit ergo angulus $fSg = g - f$; & angulus $gSh = h - g$: atque per theoriam ſolis dabuntur diſtantiæ Sf, Sg, & Sh. Repræſentet ſcilicet tabula planum eclipticæ, ſitque in prima obſervatione longitudo cometæ (ſeu planetæ) obſervata $=$ F, quam recta $f\zeta$ repræſentet: in obſervatione ſecunda ſit cometæ longitudo obſervata $=$ G, per rectam $g\eta$ repræſentata; & in obſervatione tertia ſit longitudo cometæ $=$ H, quam recta $h\theta$ exhibeat. Denique in obſervatione prima ſit Fig. 5.

ſit

latitudo cometæ $= \zeta$, in secunda $= \eta$, in tertia $= \theta$. Ex longitudinibus ergo observatis erunt anguli

$$S f \zeta = F - f, \quad S r \eta = G - f; \quad fmg = G - F$$
$$S g \eta = G - g; \quad S p \zeta = F - g; \quad gnh = H - G$$
$$S h \theta = H - h; \quad S s \theta = H - g;$$
$$S q y = G - h;$$

ducatur fx ipsi gm & $g\eta$ ipsi hn parallela, erit ob datos in triangulo Sfx omnes angulos cum latere Sf;

$$Sx = \frac{\sin(G-f)}{\sin(G-g)} Sf; \; fx = \frac{\sin(g-f)}{\sin(G-g)} Sf; \; \& \, gx = \frac{\sin(G-f)}{\sin(G-g)}$$

$Sf - Sg$. Deinde ob datos in triangulo fpx angulos cum latere Sx erit $fp = \frac{\sin(G-g)}{\sin(F-g)} fx = \frac{\sin(g-f)}{\sin(F-g)} Sf$; $px = \frac{\sin(G-F)}{\sin(F-g)} fx$, hinc est $px : fp = \sin(G-F) : \sin(G-g) = gx : fm$, unde obtinetur $fm = \frac{\sin(G-f)}{\sin(G-F)} Sf - \frac{\sin(G-g)}{\sin(G-F)} Sg$.

Tum erit $pm = \frac{\sin(g-f)}{\sin(F-g)} Sf + fm$. Ergo $\sin(G-g) : pm = \sin(F-g) : gm$ unde fit $gm = \frac{\sin(F-g)}{\sin(G-g)} fm + \frac{\sin(g-f)}{\sin(G-g)} Sf$ seu

$$gm = \frac{\sin(G-f)\sin(F-g) + \sin(g-f)\sin(G-F)}{\sin(G-g)\sin(G-F)} Sf - \frac{\sin(F-g)}{\sin(G-F)} Sg.$$

at est $\sin(G-f)\sin(F-g) + \sin(g-f)\sin(G-F) = \sin(F-f)\sin(G-g)$

ergo $gm = \frac{\sin(F-f)}{\sin(G-F)} Sf - \frac{\sin(F-g)}{\sin(G-F)} Sg$. Simili autem modo ratiocinando reperiuntur lineæ gn & hn eritque

$fm =$

$$f\,m = \frac{\sin(G-f)}{\sin(G-F)}\,Sf - \frac{\sin(G-g)}{\sin(G-F)}\,S\,g$$

$$g\,m = \frac{\sin(F-f)}{\sin(G-F)}\,Sf - \frac{\sin(F-g)}{\sin(G-F)}\,S\,g$$

$$g\,n = \frac{\sin(H-g)}{\sin(H-G)}\,Sg - \frac{\sin(H-h)}{\sin(H-G)}\,S\,h$$

$$h\,n = \frac{\sin(G-g)}{\sin(H-G)}\,Sg - \frac{\sin(G-h)}{\sin(H-G)}\,S\,h.$$

Inventis jam punctis m & n, quibus ad sequentem calculum erit opus, sit G locus cometæ verus in secunda observatione, unde perpendicularis demittatur in eclipticam $G\eta$: ac ponatur ejus a terra distantia $= r$, erit ob latitudinem observatam $= \eta$, intervallum $g\,\eta = r \cos \eta$; & $G\,\eta = r \sin \eta$, dabitur ergo locus cometæ ad eclipticam relatus η, hincque lineæ $m\,\eta = g\,\eta - g\,m$; & $n\,\eta = g\,\eta - g\,n$. Ex η ducatur recta ad solem $S\,\eta$, atque demisso ex S in $g\,\eta$ perpendiculo SM, erit $SM = Sg \sin(G-g)$ & $g\,M = Sg \cos(G-g)$: hinc erit $M\eta = g\eta - g\,M$, & $\tang \eta SM = \frac{\eta M}{SM} = \cot S\eta M$; atque $S\eta = \frac{SM}{\sin S\eta M}$. Deinde reperietur latitudo heliocentrica, erit enim $\tang GS\eta = \frac{G\eta}{S\eta}$ & distantia a sole $GS = \frac{G\eta}{\sin GS\eta} = \frac{S\eta}{\cos GS\eta}$. Sint nunc F & H loca cometæ in prima & tertia observatione, unde perpendicula in planum eclipticæ demissa cadere debent in rectas $f\,\zeta$ & $h\,\theta$, eritque

ducta corda FH, a radio SG in O secto, FO: HO $= \alpha$: β, & erit sagitta GO $= \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 \cos(\beta-\alpha)\tau}$ (44). Demittatur quoque ex O perpendiculum Oo in planum eclipticæ, quod incidet in rectam Sη; eritque ηo: GO $=$ Sη: SG; ideoque $\eta o =$ GO cos GSη. Cum igitur sit $\zeta o \theta$ corda projectionis orbitæ in ecliptica, ob $\theta o : \zeta o =$ HO: FO $= \beta : \alpha$, per punctum o duci debet recta $\zeta o \theta$ intra crura $m\zeta$ & $n\theta$, ita ut sit $\zeta o : \theta o = \alpha : \beta$, hoc est in ratione data. Ad hoc efficiendum in Sη producta capiatur oi ut sit $lo : oi = \alpha$: β, & per i ducatur ipsi $m\zeta$ parallela $i\theta$, cujus cum $n\theta$ intersectio θ dabit positionem cordæ $\zeta o \theta$ quæsitam. Sequenti autem modo per calculum puncta ζ & θ determinabuntur. Sit ang. Sη M $= \mu$, qui cum detur erit in triangulo $m\eta l$, sin (G$-$F$+\mu$): $m\eta =$ sin (G$-$F): $\eta l =$ sin μ: ml erit ergo

$$\eta l = \frac{\sin (G-F)}{\sin (G-F+\mu)} m\eta \ \& \ ml = \frac{\sin \mu}{\sin (G-F+\mu)} m\eta \text{ hinc}$$

ob $lo = \eta l - \eta o$ dabitur lo: eritque $oi = \frac{\beta}{\alpha} lo$ & li

$= \frac{(\alpha+\beta)}{\alpha} lo = \lambda\theta$, ducta $\lambda\theta$ parallela ipsi li. Nunc consideretur triangulum $\theta k \lambda$, eritque sin (H $-$ F): $\lambda\theta =$ sin (G $-$ F $+ \mu$): $k\theta =$ sin ($\mu -$ H $+$ G): $k\lambda$; unde fit

$$k\theta = \frac{\sin (\mu + G - F}{\sin (H-F)} \lambda\theta; \ \& k\lambda = \frac{\sin (\mu - H + G)}{\sin (H-F)} \lambda\theta; \text{ sicque}$$

cognoscun-

cognoſcentur puncta θ & λ; ob $km = \frac{\text{ſin}\,(H-G)}{\text{ſin}\,(H-F)}\,mn$ & $nk = \frac{\text{ſin}\,(G-F)}{\text{ſin}\,(H-F)}\,mn$; eritque $l\lambda = ml + km - k\lambda$. Deinde fiat $\beta : \alpha = l\lambda : l\zeta$, eritque $l\zeta = \frac{\alpha}{\beta}\,l\lambda$, ideoque & punctum ζ eſt repertum. Quia ergo in triangulo $\zeta k\vartheta$ dantur latera $k\zeta$ & $k\theta$ cum angulo $\zeta k\theta = H - F$, reperietur latus $\zeta\theta$ cum angulo $k\zeta\theta$, unde erit angulus $So\zeta = 180° - k\zeta\theta - \mu - G + F$; ergo & latus $\zeta\theta$ & ejus poſitio erit cognita. Tum vero ex latitudinibus obſervatis erit $F\zeta = f\zeta$ tang ζ, & $H\theta = h\vartheta$ tang ϑ; ex quo & cordæ F H poſitio innoteſcit. Producatur corda HF, donec in plano eclipticæ cum $\vartheta\zeta$ in N occurrat, eritque recta SN interſectio orbitæ cometæ & eclipticæ, ideoque linea nodorum; & in N quidem erit nodus aſcendens, ſi latitudines obſervatæ fuerint boreales, & loca cometæ ita ſint diſpoſita, ut figura repræſentat. Ad punctum N inveniendum, fiat $H\vartheta - F\zeta : \zeta\vartheta = H\vartheta : \vartheta N$, unde fit $\vartheta N = \frac{\zeta\vartheta}{H\vartheta - F\zeta} . H\vartheta$; & $\frac{H\vartheta - F\zeta}{\zeta\vartheta}$ dabit anguli $HN\vartheta$ tangentem. Deinde in triangulo SoN ob data latera So & No cum angulo $SoN = So\zeta$, reperientur anguli NSo, SNo, cum latere SN. Hinc primum ſi angulus $NS\eta + \eta Sg$ a longitudine terræ in obſervatione media, quæ eſt $= 6^s + g$, ſubtrahatur, remanebit longitudo heliocentrica nodi N. Tum ex o in SN demittatur perpendiculum oP, ductaque recta OP angulus OPo monſtrabit in-

clinationem orbitæ cometæ ad eclipticam, Erit vero $oP = No.\sin SNo$; & $Oo = No.\mathrm{tang}\, HN\vartheta$, unde erit $\mathrm{tang}\, OPo = \frac{\mathrm{tang}\, HN\vartheta}{\sin SNo} = \frac{H\vartheta - F\zeta}{\zeta\vartheta.\sin SNo}$. Deinde erit $\cos SNH = \frac{NP}{NO} = \frac{NP}{No}.\frac{No}{NO} = \cos SNo.\cos HN\vartheta$. Denique invenitur $NF = \frac{F\zeta}{\sin HN\vartheta}$; & $HN = \frac{H\vartheta}{\sin HN\vartheta}$, atque hinc in plano orbitæ cometæ SNFH dabuntur latus SN; angulus SNH, & latera NF & NH, unde reperientur anguli NSF & NSH cum lateribus SF & SH; ideoque dabuntur duæ cometæ a sole distantiæ FS & HS, una cum angulo FSH & tempore $= \alpha + \beta$ dier. quo cometa ex F in H pervenit, ex quibus orbita cometæ invenietur per probl. V. Q. E J.

Coroll. 1.

48. Quo igitur hæc orbitæ cometæ vel planetæ determinatio magis sit exacta; necesse est ut observationes non nimis à se invicem distent, atque ut ratio inter tempora $\alpha : \beta$ proxime sit ratio æqualitatis. Tum vero hoc maxime requiritur, ut observationes summa cura sint institutæ atque ut distantia cometæ a terra in observatione media a veritate quam minimum aberret.

Coroll. 2.

48. Quando longitudo cometæ in observatione media $g\eta$ in ipsam longitudinem solis gS incidit, tum angulus $S\eta g$ sit vel

fit vel nullus vel 180°, hocque adeo casu calculus non mediocriter contrahitur.

Scholion 1.

50. Operatio hæc vel per accuratam descriptionem figuræ super charta satis magna institui potest, vel, quod magis est suadendum, per calculum trigonometricum; quemadmodum ergo hunc calculum quam brevissime institui oporteat prætermisso omni ratiocinio, hic ob oculos ponamus, quo commodius eo ad orbitas cometarum investigandas uti liceat. Notentur igitur primum quæ sunt per observationes data:

Tempus elapsum ab Observatione I ad II $= \alpha$ dieb.

Tempus elapsum ab Observatione II ad III $= \beta$ dieb.

Observat.	Long.Solis	Dist.☉ a ♁	Long.Com.	Lat.Com.
I	f	Sf	F	ζ
II	g	Sg	G	η
III	h	Sh	H	θ

Hinc definiantur anguli

$$fmg = \zeta m\eta = G - F$$
$$gnh = \eta n\theta = H - G$$
$$fkh = \zeta k\theta = H - F$$

itemque

$$Sf\zeta = F - f;\quad Sr\eta = G - f;\quad Ss\theta = H - g$$
$$Sg\eta = G - g;\quad Sp\zeta = F - g;\quad Sq\eta = G - h$$
$$Sh\theta = H - h;$$

Ex his reperitur

$$fm = \frac{\sin Sr\eta}{\sin fmg} Sf - \frac{\sin Sg\eta}{\sin fmg} Sg$$

$$gm = \frac{\sin Sf\zeta}{\sin fmg} Sf - \frac{\sin Sp\zeta}{\sin fmg} Sg$$

$$g\,n = \frac{\sin S\,s\,\theta}{\sin g\,n\,b}\, S\,g - \frac{\sin S\,b\,\theta}{\sin g\,n\,b}\, S\,b$$

$$b\,n = \frac{\sin S\,g\,\eta}{\sin g\,n\,b}\, S\,g - \frac{\sin S\,q\,\eta}{\sin g\,n\,b}\, S\,b$$

unde $mn = gm - gn$; & $mk = \frac{\sin g n b}{\sin f k b}\, mn$; $nk = \frac{\sin f m g}{\sin f k b}\, mn$

Sit in observatione II distantia cometæ a terra $= r$ erit $G\eta = r \sin \eta$; $g\eta = r \cos \eta$; $m\eta = g\eta - gm$; $n\eta = g\eta - gn$. Porro est $\mathrm{tang}\,\frac{1}{2}\,(g\eta S - g S \eta) = \frac{gS - g\eta}{gS + g\eta} \cot \frac{1}{2} S g \eta$: unde erit $g\eta S = \frac{1}{2}\,(g\eta S - gS\eta) + 90 - \frac{1}{2} S g\eta$; & $gS\eta = 90 - \frac{1}{2} S g\eta - \frac{1}{2}(g\eta S - gS\eta)$

atque $S\,\eta = \frac{\sin S g \eta}{\sin g \eta S}\, S g.$

Deinde est $\mathrm{tang}\, GS\eta = \frac{G\eta}{S\eta}$; & $SG = \frac{G\eta}{\sin GS\eta}$.

Ponatur angulus $1774\frac{988''}{1000} = \tau$, ut sit $l\tau = 3,2489776$ & quærantur anguli $\alpha\tau$ & $\beta\tau$; sitque $c = 100000$; erit $GO = \frac{2c^3 \sin \alpha\tau . \sin \beta\tau}{SG^2 \cos(\beta - \alpha)\tau}$; & $\eta o = GO \cos GS\eta$; $So = S\eta - o\eta$

Deinde est ang. $\zeta l o = g\eta S + \zeta m \eta$; & $l\eta = \frac{\sin \zeta m \eta}{\sin \zeta l o}\, m\eta$

& $ml = \frac{\sin g \eta S}{\sin \zeta l o}\, m\eta$: indeque $lo = l\eta - \eta o$, ex hisque

$\lambda\theta = \frac{\alpha + \beta}{\alpha}\, lo$; $k\theta = \frac{\sin \zeta l o}{\sin \zeta k \theta}\, \lambda\vartheta$; & (ob $k\vartheta\lambda = \zeta l o - \zeta k \vartheta$)

$k\lambda =$

$k\lambda = \frac{\sin k\vartheta\lambda}{\sin \zeta k\vartheta}\lambda\vartheta$, unde $l\lambda = ml + km - k\lambda$, $l\zeta = \frac{\alpha}{\beta} l\lambda$ & $k\zeta = km + ml + l\zeta$, atque tang $\frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta) = \frac{k\zeta - k\vartheta}{k\zeta + k\vartheta}$ cot $\frac{1}{2}\zeta k\vartheta$, & $k\vartheta\zeta = 90 - \frac{1}{2}\zeta k\vartheta + \frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta)$; $k\zeta\vartheta = 90 - \frac{1}{2}\zeta k\vartheta - \frac{1}{2}(k\vartheta\zeta - k\zeta\vartheta)$ atque $\vartheta\zeta = \frac{\sin \zeta k\vartheta}{\sin k\zeta\vartheta} k\vartheta$; unde $S o \zeta = 190° k\zeta\vartheta - \zeta l o$. His inventis erit $f\zeta = fm + ml + l\zeta$: $b\vartheta = bn + nb + k\vartheta$; hinque $F\zeta = f\zeta$ tang ζ & $H\vartheta = b\vartheta$ tang ϑ, ac porro tang $HN\vartheta = \frac{H\vartheta - F\zeta}{\zeta\vartheta}$ & $\vartheta N = \frac{H\vartheta}{\text{tang } HN\vartheta}$; ob $\vartheta o = \frac{\beta}{\alpha + \beta}\zeta\vartheta$, erit $No = N\theta - \theta o$. Deinceps est tang $\frac{1}{2}(SNo - oSN) = \frac{oS - oN}{oS + oN}$ cot $\frac{1}{2} So\zeta$ & $SNo = 90 - \frac{1}{2} So\zeta + \frac{1}{2}(SNo - oSN)$; $oSN = 90 - \frac{1}{2} So\zeta - \frac{1}{2}(SNo - oSN)$ & $SN = \frac{\sin So\zeta}{\sin oSN} No$, unde longitudo heliocentrica nodi $N = 6^s + g - gS\eta - oSN$, & inclinationis orbitæ ad Eclipticam tang $= \frac{\text{tang } HN\theta}{\sin SNo}$. Deinde est cos $SNH = \cos SNo . \cos HN\vartheta$ & $FN = \frac{F\zeta}{\sin HN\theta}$; $HN = \frac{H\theta}{\sin HN\theta}$. Denique fiet tang $\frac{1}{2}(NFS - NSF) = \frac{SN - NF}{SN + NF}$ cot $\frac{1}{2} SNH$; $NFS = 90 - \frac{1}{2} SNH + \frac{1}{2}(NFS - NSF)$ $NSF = 90 - \frac{1}{2} SNH - \frac{1}{2}(NFS - NSF)$ & $SF = \frac{\sin SNH}{\sin NFS} SN$; &

tang

$\text{tang}\,\frac{1}{2}\,(\text{NHS}-\text{NSH}) = \frac{\text{SN}-\text{NH}}{\text{SN}+\text{NH}}\,\text{cot}\,\frac{1}{2}\,\text{SNH}$; unde $\text{NHS} = 90 - \frac{1}{2}\text{SNH} + \frac{1}{2}(\text{NHS}-\text{NSH})$ & $\text{NSH} = 90 - \frac{1}{2}\,\text{SNH} - \frac{1}{2}(\text{NHS}-\text{NSH})$, & $\text{SH} = \frac{\text{sin SNH}}{\text{sin NHS}}\,\text{SN}$ atque $\text{FSH} = \text{NSH} - \text{NSF}$.

Scholion. 2.

51. Inventis duabus distantiis FS & HS cum angulo FSH & tempore inter hæc loca elapso $= \alpha + \beta$ dier. orbita sequenti calculo definietur. Sit

distantia $\text{FS} = y$, tempus $\alpha + \beta = \text{T}$
distantia $\text{HS} = z$, & ang $\text{FSH} = \phi$

ob $m = 271989,735$ & $lm = 5,4345525139$ erit orbitæ latus rectum $b = \left(\frac{yyzz}{4m^2\text{T}^2} + \frac{1}{3}\sqrt{yz}\right)(\text{sin}\,\phi)^2$. Sit distantia perihelii a sole $= a$; & anomalia vera loci F sit $= v$, erit $\text{tang}\,v = \text{cot}\,\phi - \frac{(b-z)\,y}{(b-y)\,z\,\text{sin}\,\phi}$, & $a = \frac{by\,\text{cos}\,v}{b-y+y\,\text{cos}\,v}$.

Sumatur $\text{tang}\,\frac{1}{2}\,\omega = \frac{\sqrt{(2a-b)}}{\sqrt{b}}\,\text{tang}\,\frac{1}{2}\,v$, si quidem fuerit $2a > b$ & orbita ellipsis: eritque tempus, quo cometa a perihelio ad locum F pervenit, in diebus expressum $= \frac{a^3}{2m\,(2a-b)^{\frac{3}{2}}}\left(\omega - \frac{(b-a)}{a}\,\text{sin}\,\omega\right)$, sin autem fuerit $b > 2a$ & curva hyperbola, sumatur $\text{tang}\,\frac{1}{2}\,\omega = \frac{\sqrt{(b-2a)}}{\sqrt{b}}\,\text{tang}\,\frac{1}{2}\,v$, & erit tempus

quo

quo cometa a perihelio ad F pervenit in diebus $= \frac{a^3}{2m(b-a)^{\frac{3}{2}}}$ $\left(\frac{b-a}{a} \text{tang}\, \omega - l\, \text{tang}\,(45^\circ + \frac{1}{2}\omega)\right)$. At si orbita cometæ fuerit vel parabola vel ad eam proxime accedat, ponatur tang $\frac{1}{2}v = t$; & $\delta = 2a - b$ & $n = \frac{\delta}{b} = \frac{2a-b}{b}$, eritque tempus, quo cometa a perihelio ad F pervenit, in diebus expressum $=$

$$\frac{aa}{m\sqrt{b}}\left(t + \tfrac{1}{3}t^3 \begin{matrix} - \tfrac{2}{5}n \\ + \tfrac{3}{5}nn \end{matrix} t^5 \begin{matrix} + \tfrac{3}{7}nn \\ - \tfrac{4}{7}n^3 \end{matrix} t^7 \begin{matrix} - \tfrac{4}{9}n^3 \\ + \tfrac{5}{9}n^4 \end{matrix} t^9 \ \&c.\right)$$

Cum igitur tempus constet, quo cometa in F est versatus, simul temporis momentum habebitur, quo cometa per perihelium transiit. Denique si ab anomalia vera $= v$ loci F subtrahatur angulus FSN, habebitur distantia nodi N a perihelio. Ceterum hoc problema parum utilitatis habere videatur, cum nunquam parallaxis cometæ tam exacte definiri queat, quam ad hoc institutum opus est; at vero summus usus hujus resolutionis perspicietur in problemate sequenti.

Problema XII.

52. Ex aliquot observationibus cometæ ejus orbitam veram determinare.

Solutio.

Ex observationibus cometæ, quibus vel ejus distantiæ a stellis fixis, vel altitudines meridianæ sunt mensuratæ, elician-

tur ejus longitudines ac latitudines geocentricæ, & ad unamquamque obſervationem notetur temporis momentum, quo eſt facta, ſecundum tempus medium. Ex his cometæ locis in ordinem diſpoſitis eligantur tria non nimis a ſe invicem remota, ita ut media ratione temporis fere medium inter jaceat inter extremas. Tum pro lubitu in obſervatione media fingatur cometæ diſtantia a terra, ac per problema præcedens ex his tribus locis & diſtantia ficta determinetur orbita cometæ, quæ erit vera, ſi diſtantia illa ficta cum vera conveniat, contra autem falſa. Sumatur ergo obſervatio quædam quarta a tribus illis electis longiſſime remota, atque ex orbita eruta ad ejus tempus computetur locus cometæ geocentricus, qui ſi cum obſervato congruat, docebit orbitam erutam eſſe veram, ſin minus congruat, eſſe falſam. Fingantur ergo ſimul duæ tresve diſtantiæ, quas cometa in obſervatione media habuerit, inter ſe diverſæ, & ex qualibet definiatur orbita cometæ, & diſpiciatur quantum quæque ab obſervatione quarta diſcrepet. Atque hinc etiamſi nulla orbita hoc modo inventa ſit vera, tamen facile intelligetur, quænam ad veritatem propius accedat; atque adeo, ſi diſſenſus non fuerit admodum magnus, per interpolationem vera cometæ orbita erui poterit. Sin autem diſſenſus ſit adhuc nimis magnus, tum ſaltem diſtantia cometæ in obſervatione media propius cognoſcetur; quare fingendis aliquot

novis

novis diſtantiis a veritate minus abhorrentibus, ſi ex iis pari modo orbitæ definiantur, & cum obſervatione quarta conferantur, per interpolationem ſatis exacte vera orbita cometæ concludi poterit. Q. E. J.

Coroll. 1.

53. Quoniam prima fictio ideo tantum fit, ut orbita cometæ veræ tantum propinqua obtineatur, non opus eſt, ut calculus omni cura inſtituatur, ſed ſufficiet per conſtructionem geometricam ex diſtantiis fictis orbitam eruere.

Coroll. 2.

54. Eodem modo poſſent etiam planetarum orbitæ investigari, atque ex quatuor obſervationibus definiri. At quoniam in planetis per continuas obſervationes tempora periodica & lineæ nodorum facile determinantur, his cognitis certior aperitur via ad eorum orbitas definiendas, quam potius ſequi convenit.

Scholium.

55. Hujus methodi utilitas aptiſſime per exemplum docebitur, ſimulque patebit, quemadmodum commodiſſime calculum ſatis prolixum inſtitui oporteat. Hunc in finem cometam, qui circa finem A. 1680 & initium ſequentis anni

apparuit, potiſſimum investigari conveniet, quippe quo vix ullus alius majori cura atque opportunitate non ſolum eſt obſervatus, ſed etiam ejus orbita per calculum determinata. Cum igitur hujus cometæ & obſervationes per quadrimeſtre fere temporis ſpatium habeantur accuratiſſimæ, & ipſa ejus orbita Virorum Celeberrimorum Neutoni & Halleji ſtudiis ſit determinata, etiamſi actum agere videar, tamen hoc exemplo mea methodus non ſolum maxime illustrabitur, ſed etiam ex conſenſu fortiſſime confirmabitur. Obſervationes autem hujus cometæ in Principiis Math. Philoſophiæ Neutonianæ recenſentur, unde eas hic depromam.

Cometæ, qui circa finem anni 1680.

& initium anni 1681. apparuit, loca obſervata ad tempus medium ſtili veteris & meridianum Londinenſem reducta.

	d. h. m.	Longitudo	Latitudo	
A.1680.M.Nov.	16,17,10	6ˢ,8°,0′	0°,44,′	auſtr.
	17,17,10	6,12,52	1, 0	
	18,21,44	6,18,40	1, 18	
	19,17,10	6,22,48	1, 30	
	20,17,—	6,27,52	1, 45	
	21,17,—	7, 2,56	1, 58	
	23,17,15	7,12,58	2, 20	
	24,17,30	7,17,53	2, 29	
M.Dec.	12 4,46,0″	9, 6,31,21″	8,26,0″	bor.
	21, 6,36,59	10, 5, 7,38	21,45,30	
	24, 6,17,52	10,18,49,10	25,23,24	
	26, 5,20,44	10,28,24, 6	27, 0,57	
	29, 8, 3, 2	11,13,11,45	28,10, 5	
	30, 8,10,26	11,17,39, 5	28,11,12	
A.1681. M. Jan.	5, 6, 1,38	0, 8,49,10	26,15,26	
	9, 7, 0,53	0,18,43,18	24,12,42	
	10, 6, 6,10	0,20,40,57	23,44, 0	
	13, 7, 8,55	0,25,59,34	22,17,36	
	25, 7,58,42	1, 9,35,48	17,56,54	
	30, 8,21,53	1,13,19,36	16,40,57	
M. Febr.	2, 6,34,51	1,15,13,48	16, 2, 2	
	5, 7, 4,41	1,16,59,52	15,27,23	
	25, 8,41,—	1,26,18,17	12,46,54	
	27, 8,26,—	1,27, 4,24	12,36,12	
M.Mart.	1,11,10,—	1,27,53, 6	12,24,52	
	2, 8,10,—	1,28,12,27	12,20, 0	
	5,11,39,—	1,29,20,51	12, 3,30	
	9, 8,38,—	2, 0,43, 3	11,45,53	

Investigatio Orbitæ hujus Cometæ.

Eligantur tres sequentes observationes Mense Januario A. 1681. factæ

Tempus	Long. Solis	Dist. Sol. a Ter.	Long. Comet.	Lat. Cometæ
$5^d, 6^h, 1', 38''$	$9^s, 26°, 22', 18''$	9 8 3 6 3, 5	$0^s, 8°, 49', 10''$	$26°, 15', 26''$
9, 7, 0, 53	10, 0, 29, 2	9 8 4 0 7, 0	0, 18, 43, 18	24, 12, 42
13, 7, 8, 55	10, 4, 33, 20	9 8 4 5 8, 8	0, 25, 59, 34	22, 17, 36

Hinc erit

$$\alpha = 4^d,\ 0^h,\ 59',\ 15'' = 4,\ 0411;\ l\alpha = 0,\ 6064996$$

$$\beta = 4^d,\ 0^h,\ 8',\ 2'' = 4,\ 0055;\ l\beta = 0,\ 6026567$$

$$\alpha + \beta = 8,\ 0466;\ \&\ l(\alpha + \beta) = 0,\ 9056124$$

$$f = 9^s, 26°, 22', 18'';\ F = 0^s, 8°, 49', 10'';\ \zeta = 26°, 15', 26''$$

$$g = 10,\ 0,\ 29,\ 2;\ G = 0,\ 18, 43,\ 18;\ \eta = 24,\ 12,\ 42$$

$$h = 10,\ 4,\ 33,\ 20;\ H = 0,\ 25, 59,\ 34;\ \theta = 22,\ 17,\ 36$$

$$f\,m\,g = \zeta\,m\,\eta = G - F = 9°,\ 54',\ 8''$$

$$g\,n\,h = \eta\,n\,\theta = H - G = 7°,\ 16', 16''$$

$$f\,k\,h = \zeta\,k\,\theta = H - F = 17,\ 10,\ 24$$

$$S\,f\,\zeta = F - f = 72,\ 26,\ 52$$

$$S\,g\,\eta = G - g = 78,\ 14,\ 16$$

$$S\,h\,\theta = H - h = 81,\ 26,\ 14$$

$$S\,r\,\eta = G - f = 82,\ 21,\ 0$$

$$S\,p\,\zeta = F - g = 68,\ 20,\ 8$$

$$S\,s\,\theta = H - g = 85,\ 30,\ 32$$

$$S\,q\,\eta = G - h = 74,\ 9,\ 58$$

$$Sf = 98363, 5;\ l\,Sf = 4, 9928340$$

$$Sg = 98407, 0;\ l\,Sg = 4, 9930260$$

$$Sh = 98458, 8;\ l\,Sh = 4, 9932545$$

Jam sequentes calculi instituantur. $l\,S\,f$

$$l\,Sf = 4,\ 9928340$$
$$\text{subt.}\ l\sin fmg = 9,\ 2354458$$
$$5,\ 7573882$$
$$l\sin Sr\eta = 9,\ 9961174$$
$$l\sin Sf\zeta = 9,\ 9792946$$
$$l\frac{\sin Sr\eta}{\sin fmg}Sf = 5,\ 7535056$$
$$l\frac{\sin Sf\zeta}{\sin fmg}Sf = 5,\ 7366828$$

$$l\,Sg = 4,\ 9930260$$
$$l\sin gnh = 9,\ 1023116$$
$$5,\ 8907144$$
$$l\sin Ss\theta = 9,\ 9986644$$
$$l\sin Sg\eta = 9,\ 9907836$$
$$l\frac{\sin Ss\theta}{\sin gnh}Sg = 5,\ 8893788$$
$$l\frac{\sin Sg\eta}{\sin gnh}Sg = 5,\ 8814980$$

$$l\,Sg = 4,\ 9930260$$
$$l\sin fmg = 9,\ 2354458$$
$$5,\ 7575802$$
$$l\sin Sg\eta = 9,\ 9907836$$
$$l\sin Sp\zeta = 9,\ 9681848$$
$$l\frac{\sin Sg\eta}{\sin fmg}Sg = 5,\ 7483638$$
$$l\frac{\sin Sp\zeta}{\sin fmg}Sg = 5,\ 7257650$$

$$l\,Sh = 4,\ 9932545$$
$$l\sin gnh = 9,\ 1023116$$
$$5,\ 8909429$$
$$l\sin Sh\theta = 9,\ 9951318$$
$$l\sin Sq\eta = 9,\ 9832008$$
$$l\frac{\sin Sh\theta}{\sin gnh}Sh = 5,\ 8860747$$
$$l\frac{\sin Sq\eta}{\sin gnh}Sh = 5,\ 8741437$$

$$\frac{\sin Sr\eta}{\sin fmg}Sf = 566898,\ 9$$
$$\frac{\sin Sg\eta}{\sin fmg}Sg = 560226,\ 8$$
$$fm = 6672,\ 1$$
$$\frac{\sin Sf\zeta}{\sin fmg}Sf = 545359,\ 4$$
$$\frac{\sin Sp\zeta}{\sin fmg}Sg = 531820,\ 4$$
$$gm = 13539,\ 0$$
$$gn = 5875,\ 0$$
$$mn = 7664,\ 0$$

$$\frac{\sin Ss\theta}{\sin gnh}Sg = 775137,\ 7$$
$$\frac{\sin Sh\theta}{\sin gnh}Sh = 769262,\ 7$$
$$gn = 5875,\ 0$$
$$\frac{\sin Sg\eta}{\sin gnh}Sg = 761198,\ 5$$
$$\frac{\sin Sq\eta}{\sin gnh}Sh = 748417,\ 0$$
$$hn = 12781,\ 5$$
$$l\,mn = 3,\ 8844555$$
$$l\sin fkh = 9,\ 4702096$$
$$4,\ 4142459$$

mk	=	3285, 168	l sin gnb	=	9, 1023116
nk	=	4463, 666	l sin fmg	=	9, 2354458
			$l\,mk$	=	3, 5165575
			$l\,nk$	=	3, 6496917

Ut jam labori in ſcrutando valore diſtantiæ $gG = r$ vero proximo parcamus, conſulamus Theoriam Neutoni, quæ pro hoc tempore exhibet diſtantiam cometæ a ſole = 1 1 0 9 7 0, unde elicitur diſtantia cometæ a terra = 7 2 7 4 7. Hanc obrem pro r aſſumam hos duos valores 7 2 7 0 0 & 7 2 8 0 0.

	Sit ergo r	=	72700	72800
	lr	=	4, 8615344	4, 8621314
	add { l ſin η	=	9, 6128990	9, 6128990
	{ l coſ η	=	9, 9600122	9, 9600122
	$l\,G\eta$	=	4, 4744334	4, 4750304
	$l\,g\eta$	=	4, 8215466	4, 8221436
	$g\eta$	=	66305, 05	66396, 26
$Sg\eta = 78°, 14', 16''$	gm	=	13539, 0	13539, 0
$\frac{1}{2}Sg\eta = 39, 7', 8$	gn	=	5875, 0	5875, 0
Compl. $= 50, 52', 52''$	$m\eta$	=	52766, 05	52857, 26
	$n\eta$	=	60430, 05	60521, 26
	Sg	=	98407, 0	98407, 0
	$g\eta$	=	66305, 05	66396, 26
	$Sg + g\eta$	=	164712, 05	164803, 26
	$Sg - g\eta$	=	32101, 95	32010, 74
	$l(Sg - g\eta)$	=	4, 5065314	4, 5052957
	$l(Sg + g\eta)$	=	5, 2167254	5, 2169658
			9, 2898060	9, 2883299
	l tang $(90 - \frac{1}{2}Sg\eta)$	=	10, 0897890	10, 0897890
	l tang $\frac{1}{2}(g\eta S - gS\eta)$	=	9, 3795950	9, 3781189

$\frac{1}{2}(g\eta S - gS\eta)$

$\frac{1}{2}(g\eta S - gS\eta) =$	13°,28′,38″	13°,25′,59″
$90 - \frac{1}{2} Sg\eta =$	50, 52, 52	50, 52, 52′
$g\eta S =$	64, 21 30	64, 18, 51
$gS\eta =$	37, 24, 14	37, 26, 53
Ad $l\,Sg =$	4, 9930260	4, 9930260
add l sin $Sg\eta =$	9, 9907836	9, 9907836
	14, 9838096	14, 9838096
ſubtr. l ſin $g\eta S =$	9, 9549744	9, 9548136
$l\,S\eta =$	5, 0288352	5, 0289960
$a \quad lG\eta =$	4, 4744334	4, 4750304
l tang $GS\eta =$	9, 4455982	9, 4460344
$GS\eta =$	15°,35′,20″	15°,36′,14″
A $lG\eta =$	4, 4744334	4, 4750304
ſubtr. l ſin $GS\eta =$	9, 4293210	9, 4297284
$lSG =$	5, 0451124	5, 0453020
$l\tau =$	3, 2489776	
$l\alpha =$	0, 6064996	
$l\beta =$	0, 6026567	
$l\alpha\tau =$	3, 8554772	
$l\beta\tau =$	3, 8516343	
$\alpha\tau =$	7169″,3 =	1°,59′,29″
$\beta\tau =$	7106″,1 =	1°,58′,26″
$(\alpha-\beta)\tau =$	63, 2 =	1′, 3″
l ſin $\alpha\tau =$	8, 5409422	
l ſin $\beta\tau =$	8, 5371103	
$l\,2c^3 =$	15, 3010300	
	32, 3790825	
ſubt. l coſ$(\alpha-\beta)\tau =$	10, 0000000	
	12, 3790825	12, 3790825
ſubtr 2 $l\,SG =$	10, 0902248	10, 0906040

$lGO =$

lGO =	2, 2888577	2, 2884785
add. l coſ G Sη =	9, 9837231	9, 9836924
$l\eta o$ =	2, 2725808	2, 2721709
ηo =	187,32	187,14
ab Sη =	106864,9	106904,5
So =	106677,6	106717,4
Ang. $g\eta$S =	64°,24′,30″	64°,18′,51″
$\zeta m\eta$ =	9, 54, 8	9, 54, 8
ζlo =	74, 15, 38	74, 12, 59″
$lm\eta$ =	4, 7223546	4, 7231046
ſubtr. l ſin ζlo =	9, 9834031	9, 9833086
	4, 7389515	4, 7397960
add { l ſin $\zeta m\eta$ =	9, 2354458	9, 2354458
l ſin $g\eta$S =	9, 9549744	9, 9548136
l $l\eta$ =	3, 9743973	3, 9752418
lml =	4, 6939259	4, 6946096
Ergo $l\eta$ =	9427, 52	9445, 87
ſubtr. ηo =	187, 32	187, 14
lo =	9240, 20	9258, 73
l lo =	3, 9656814	3, 9665515
add $l\,\frac{(\alpha+\beta)}{\alpha}$ =	0, 2991128	0, 2991128
erit $l\lambda\theta$ =	4, 2647942	4, 2656643
Ang. ζlo =	74°,15′,38′	74°,12′,59″
ſubtr. $\zeta k\theta$ =	17°,10, 24	17, 10, 24
$k\theta\lambda$ =	57 ,5 ,14	57 ,2 ,35
$l\lambda\theta$ =	4, 2647942	4, 2656643
ſubtr. l ſin $\zeta k\theta$ =	9, 4702096	9, 4702096
	4, 7945846	4, 7954547
add { l ſin ζlo =	9, 9834031	9, 9833086
l ſin $k\theta\lambda$ =	9, 9240200	9, 9238032

$lk\theta$ =

$l k\theta =$	4, 7779877	4, 7787633
$l k\lambda =$	4, 7186046	4, 7192579
$m l =$	49422, 63	49500, 50
$k m =$	3285, 17	3285, 17
$k l =$	52707, 80	52785, 67
ſubtr. $k\lambda =$	52312, 40	52391, 15
$l\lambda =$	395, 40	394, 52
$l l\lambda =$	2, 5970367	2, 5960690
add. $l \frac{\alpha}{\beta} =$	0, 0038429	0, 0038429
$l.\ l\zeta =$	2, 6008796	2, 5999119
$\frac{1}{2}\ \zeta k\theta =$	8°,35′,12″	8°,35′,12″
$90 - \frac{1}{2}\ \zeta k\theta =$	81°,24′,48″	81°,24′,48″
$k\zeta =$	3285, 17	3285, 17
$m l =$	49422, 63	49500, 50
$l\zeta =$	398, 91	398, 02
$k\zeta =$	53106, 71	53183, 69
$k\theta =$	59977, 42	60084, 63
$k\theta + k\zeta =$	113084, 13	113268, 32
$k\theta - k\zeta =$	6870, 71	6900, 94
$l\ (k\theta - k\zeta) =$	3, 8370016	3, 8389082
$l\ (k\theta + k\zeta) =$	5, 0534016	5, 0541084
	8, 7836000	8, 7847998
l tang $(90 - \frac{1}{2}\zeta k\theta) =$	10, 8210294	10, 8210294
	9, 6046294	9, 6058292
$\frac{1}{2}\ (k\zeta\theta - k\theta\zeta) =$	21°,55′,7″	21°,58′,24″
$\frac{1}{2}(k\zeta\theta + k\theta\zeta) =$	81, 24, 48	81, 24, 48
$k\zeta\theta =$	103, 19,55	103, 23,12
$k\theta\zeta =$	59, 29,41	59, 26,24
$k\zeta\theta + \zeta l o =$	177, 35,33	177, 36,11
$S o \zeta =$	2°, 24,27	2, 23,49

A $l \sin \zeta k \theta$ =	9, 4702096	9, 4702096
subtr. $l \sin k \zeta \theta$ =	9, 9881354	9, 9880368
	9, 4820742	9, 4821728
add $l\, k \theta$ =	4, 7779877	4, 7787633
$l \theta \zeta$ =	4, 2600619	4, 2609361
fm =	6672, 1	6672, 1
ml =	49422, 6	49500, 5
$l \zeta$ =	398, 9	398, 0
erit $f \zeta$ =	56493, 6	56570, 6
hn =	12781, 5	12781, 5
nk =	4463, 6	4463, 6
$k \vartheta$ =	59977, 4	60084, 6
erit $h \vartheta$ =	77222, 6	77329, 8
ad $l\, f \zeta$ =	4, 7519993	4, 7525908
add $l \tan \zeta$ =	9, 6931129	9, 6931129
$l \mathrm{F} \zeta$ =	4, 4451122	4, 4457037
$l h \vartheta$ =	4, 8877445	4, 8883470
add. $l \tan \vartheta$ =	9, 6127775	9, 6127775
$l \mathrm{H} \vartheta$ =	4, 5005220	4, 5011245
Ergo $\mathrm{H} \vartheta$ =	31660, 8	31704, 8
$\mathrm{F} \zeta$ =	27868, 4	27906, 4
$\mathrm{H}\vartheta - \mathrm{F}\zeta$ =	3792, 4	3798, 4
$l (\mathrm{H}\vartheta - \mathrm{F}\zeta)$ =	3, 5789141	3, 5796007
subtr. $l \zeta \vartheta$ =	4, 2600619	4, 2609361
$l \tan \mathrm{HN} \vartheta$ =	9, 3188522	9, 3186646
$a\, l \mathrm{H} \vartheta$ =	4, 5005220	4, 5011245
$l \mathrm{N} \vartheta$ =	5, 1816698	5, 1824599
Ang. $\mathrm{HN}\vartheta$ =	11°,46′,15″	11°,45′,57″
$\mathrm{N} \vartheta$ =	151939, 2	152215, 9

$l \zeta \vartheta$ =

$l\zeta\theta$ =	4, 2600619	4, 2609361
subt. $l\frac{\alpha+\beta}{\beta}$ =	0, 3029557	0, 3029557
$l\theta o$ =	3, 9571062	3, 9579804
θo =	9059, 5	9077, 8
No =	142879, 7	143138, 1
$\frac{1}{2} So\zeta$ =	1°,12′,13½″	1°,11′,54½″
$90-\frac{1}{2}So\zeta$ =	88°,47′,46½″	88°,48′, 5½″
So =	106677, 6	106717, 4
No =	142879, 7	143138, 1
$No+So$ =	249557, 3	249855, 5
$No-So$ =	36202, 1	36420, 7
A $l(No-So)$ =	4, 5587338	4, 5613482
subt. $l(No+So)$ =	5, 3971703	5, 3976889
	9, 1615635	9, 1636583
add l tang $(90-\frac{1}{2}So\zeta)$ =	11, 6775226	11, 6794316
l tang $\frac{1}{2}(oSN-SNo)$ =	10, 8390861	10, 8430899
$\frac{1}{2}(oSN-SNo)$ =	81°,45′,29″½	81°,49′,58″
$\frac{1}{2}(oSN+SNo)$ =	88, 47, 46 ½	88, 48, 5½
oSN =	170, 33, 16	170, 38, 3½
SNo =	7, 2, 17	6, 58, 7½
ab l sin $So\zeta$ =	8, 6233157	8, 6214086
subtr. l sin oSN =	9, 2151360	9, 2114815
	9, 4081797	9, 4099271
add. lNo =	5, 1549705	5, 1557552
lSN =	4, 5631502	4, 5656823
Ang. $gS\eta$ =	37°,24′,14″	37°,26′,53″
add oSN =	5ˢ,20, 33, 16	5ˢ,20, 38, 3
subtr.	6ˢ,27, 57, 30	6ˢ,28, 4, 56″
$a\,6^s+g$ =	16, 0, 29, 2	16, 0, 29 2

Long. Nodi Ascend.	$9^s,2°,31',32''$	$9^s,2°,24', 6''$
l tang HNθ =	9, 3188523	9, 3186645
subtr. l sin SNo =	9, 0882372	9, 0839610
l tang. Inclinat.	10, 2306151	10, 2347035
Inclin. Orbitæ ad Eclipt. =	$59°,32',38''$	$59°,46',45''$
l cos SNo =	9, 9967152	9, 9967798
add. l cos HNθ =	9, 9907700	9, 9907779
l cos SNH =	9, 9874852	9, 9875577
Ergo SNH =	$13°,41',19''$	$13°,38',59''$
a { l Fζ =	4, 4451122	4, 4457037
l Hθ =	4, 5005220	4, 5011245
subtr. l sin HNθ =	9, 3096223	9, 3094424
l FN =	5, 1354899	5, 1362613
l HN =	5, 1908997	5, 1916821
½ SNH =	$6°, 50',39\frac{1}{2}''$	$6°, 49',29\frac{1}{2}''$
90 − ½ SNH =	$83°, 9',20\frac{1}{2}''$	$83°,10, 30\frac{1}{2}''$
SN =	36572, 12	36785, 97
NF =	136612, 3	136855, 2
NF + SN =	173184, 4	173641, 2
NF − SN =	100040, 2	100069, 2
a l(NF − SN) =	5, 0001745	5, 0003004
subt. (lNF + SN) =	5, 2385087	5, 2396528
	9, 7616658	9, 7606476
add l tang (90 − ½ SNH) =	10, 9207206	10, 9219679
l tang ½ (NSF − NFS) =	10, 6823864	10, 6826155
½ (NSF − NFS) =	$78°,15',42\frac{1}{2}''$	$78°,16',4\frac{1}{3}''$
½ (NSF + NFS) =	83, 9, 20½	83, 10, 30½
NSF =	161, 25, 3	161, 26, 35
NFS =	4, 53, 38	4, 54, 26⅙
A l sin SNH =	9, 3740724	9, 3728853
subtr l sin NFS =	8, 9310033	8, 9321860

add.

	0, 4430691	0, 4406993
add. lSN =	4, 5631502	4, 5656823
lSF =	5, 0062193	5, 0063816
NH =	155202, 8	155482, 7
SN =	36572, 12	36785, 97
NH+SN =	191774, 9	192268, 7
NH − SN =	118630, 7	118696, 7
A l(NH − SN) =	5, 0741971	5, 0744387
ſubt. l(NH+SN) =	5, 2827918	5, 2839086
	9, 7914053	9, 7905301
add l tang (90 − $\frac{1}{2}$ SNH) =	10, 9207206	10, 9219679
l tang $\frac{1}{2}$ (NSH − NHS) =	10, 7121259	10, 7124980
$\frac{1}{2}$ (NSH − NHS) =	79°,1′,9$\frac{1}{6}$″	79°,1′,42$\frac{1}{2}$
$\frac{1}{2}$ (NSH+NHS) =	83, 9, 20$\frac{1}{2}$	83,10, 30$\frac{1}{2}$
NSH =	162, 10,29$\frac{2}{3}$	162, 12,13
NHS =	4, 8,11$\frac{1}{3}$	4, 8,48
A l ſin SNH =	9, 3740724	9, 3728853
ſubtr. l ſin NHS =	8, 8581311	8, 8591973
	0, 5159413	0, 5136880
Add lSN =	4, 5631502	4, 5656823
lSH =	5, 0790915	5, 0793703
NSH =	162°,10′,29$\frac{2}{3}$″	162°,12′,13″
NSF =	161, 25, 3	161, 26, 35
FSH =	0°,45′,26$\frac{2}{3}$″	0°,45′,38″
FS = y =	101442, 3	101480, 3
HS = z =	119975, 2	120052, 2
lT =	0, 9056124	0, 9056124
Φ =	0°,45′,26$\frac{2}{3}$″	0°, 45′,38″
l ſin Φ =	8, 1211936	8, 1229950
l coſ Φ =	9, 9999635	9, 9999638

l cot Φ =

$l \cot \Phi$	=	11, 8787699	11, 8769688
$l y$	=	5, 0062193	5, 0063816
$l z$	=	5, 0790915	5, 0793703
$l y z$	=	10, 0853108	10, 0857519
$l y y z z$	=	20, 1706216	20, 1715038
$l 4$	=	0, 6020600	
$l m^2$	=	10, 8691050	
$l T^2$	=	1, 8112248	
$l 4 m^2 T^2$	=	13, 2823898	13, 2823898
$l \frac{yyzz}{4m^2T^2}$	=	6, 8882318	6, 8891140
add. $2 l \sin \Phi$	=	16, 2423872	16, 2459900
		3, 1306190	3, 1351040
Pars prior	=	1350, 887	1364, 910
$l \sqrt{yz}$	=	5, 0426554	5, 0428759
ſubt. $l 3$	=	0, 4771213	0, 4771213
		4, 5655341	4, 5657546
add $2 l \Phi$	=	16, 2423872	16, 2459900
		0, 8079213	0, 8117446
pars poſt.	=	6, 426	6, 482
Ergo b	=	1357, 313	1371, 392
y	=	101442, 3	101480, 3
z	=	119975, 2	120052, 2
$y-b$	=	100085, 0	100108, 9
$z-b$	=	118617, 9	118680, 8
A $l(z-b)$	=	5, 0741502	5, 0743804
ſubtr. $l(y-b)$	=	5, 0003690	5, 0004728
		0, 0737812	0, 0739076
Add $l \frac{y}{z}$	=	9, 9271278	9, 9270113

ſubtr.

	10, 0009090	10, 0009189
subt. l sin Φ =	8, 1211936	8, 1229950
l post. subtr.	1, 8797154	1, 8779239
Pass subtrah. =	75, 80807	75, 49600
a cot Φ =	75, 64322	75, 33014
— tang v =	0, 16485	0, 16586
Ergo — v =	9°,21′,40″	9°,25′, 4″
Anom. vera loci F seu v =	170°,38′,20″	170°,34′,56″
subtr. FSN =	161°,25′, 3″	161°,26′,35″
Dist. Nodi ☊ a perihelio	9°,13′,17″	9°, 8′,21″
Ad. ly =	5, 0062193	5, 0063816
add. l — cos v =	9, 9941775	9, 9941065
l — y cos v =	5, 0003968	5, 0004881
add. lb =	3, 1326800	3, 1371615
l Num =	8, 1330768	8, 1376496
— y cos v =	100091, 41	100112, 45
add. $y - b$ =	100085, 0	100108, 9
Denom.	200176, 4	200221, 3
l Denom. =	5, 3014129	5, 3015103
la = =	2, 8316639	2, 8361393
Distantia perih. a Sole a =	678, 678	685, 708
Ergo $2a$ =	1357, 356	1371, 416
& b =	1357, 313	1371, 392
$\delta = 2a - b$ =	0, 043	0, 024

Est ergo orbita cometæ utroque casu Ellipsis maxime oblonga & parabolæ proxima;

$al\delta$ =	(−2), 6334685	(−2), 3802112
subtr. lb =	3, 1326800	3, 1371615
n = ln =	5, 5007885	5, 2430497 Charact. 10: minuatur.

$\frac{1}{2}v =$	85°,19′,10″	85°,17′,28″
$lt =$	1, 0868576	1, 0842248
$n =$	0, 0000316	0, 0000175
$lt^3 =$	3, 2605728	3, 2526744
$lt^5 =$	5, 4342880	5, 4211240
$lt^7 =$	7, 6080032	7, 5895736
$lnt^5 =$	0, 9350765	0, 6641737
$lnnt^7 =$	(−2), 6095802	(−2), 0756730
$t =$	12, 21399	12, 14017
$\frac{1}{3}t^3 =$	607, 36750	596, 4215
$t + \frac{1}{3}t^3 =$	619, 5815	608, 5616
subtr.	3, 4445	1, 8460
add.	0, 0174	0, 0051
$t + \frac{1}{3}t^3 - \frac{2}{5}nt^5 + \frac{3}{7}n^2t^7 - \&c. =$	616, 1544	606, 7207
hujus log.	2, 7896895	2, 7829888
$2\,la =$	5, 6633278	5, 6722786
subtr. $l\sqrt{b} =$	1, 5663400	1, 5685808
	4, 0969878	4, 1036978
subtr. $lm =$	5, 4345525	5, 4345525
	8, 6624353	8, 6691473
add.	2, 7896895	2, 7829888
	1, 4521248	1, 4521361
A Perih. ad F dies. =	28, 3221	28, 3228
seu	$28^d,7^h,43',50''$	$28^d,7^h,44',50''$
At est tempus in F = A 1680 Dec.	$36^d,6$, 1, 38	$39^d,9$ 1, 38
Cometa per perihelium transiit A. 1680 Decembr.	$7^d,22^h,17',48''$	$7^d,22^h,16',48''$
Distantia Perih. a Sole =	678, 678	685, 708
semilatus rectum =	1357, 313	1371, 392
Dist. Nodi ☊ a Perihelio =	9°,13′,17″	9°, 8′,21″
Longitudo Nodi ☊ =	9^s,2°,31′,32″	9^s,2°,24′, 6″
Inclinatio Orbitæ ad Eclipt.	59°,32′,38″	59°,46′,45″

Compa-

Comparentur jam hæ binæ determinationes cum obſervatione quadam quarta; quem in finem eligatur obſervatio A. 1680 die 12 Decembris facta, quæ ita ſe habebat.

A. 1680. Dec.	12^d, 4^h, 46′, 0″		
Longitudo Cometæ;	9^s, 6°, 31′, 21″		
Latitudo Cometæ bor.	8°, 26′, 0″		
Longitudo Solis	9^s, 1°, 51′, 23″		
Diſt. Solis a terra =	98275, ejus log. = 4,9924431		

Temp. propoſitum A. 1680 Mens. Dec.		$12^d, 4^h, 46', 0''$	$12^d, 4^h, 46', 0''$
ſubtr. temp. Perihelii		7, 22,17,48	7, 22,16,48″
Erit tempus T	=	4 , 6,28,12	4 , 6,29,12
& in diebus eſt T	=	4, 26958	4, 27027
l T	=	0, 6303852	0, 6304553
add $l\,m$	=	5, 4345525	5, 4345525
		6, 0649377	6, 0650078
ſubtr. $l\,\frac{a\,a}{\sqrt{b}}$	=	4, 0969878	4, 1036978
Vid. Probl. VII. $l\,n$	=	1, 9679499	1, 9613100
& n	=	92, 88592	91, 47660
$\frac{1}{2}\,n$	=	46, 44296	45, 73830
Ergo $\frac{3}{2}\,n$	=	139, 32888	137, 21490
$l\,\frac{3}{2}\,n$	=	2, 1440412	2, 1374012
$2\,l\,\frac{3}{2}\,n$	=	4, 2880824	4, 2748024
$\frac{9}{4}\,n\,n$	=	19413, 55	18827, 93
ad 1		19414, 55	18828, 93
$l\,(\frac{9}{4}\,n\,n + 1)$	=	4, 2881271	4, 2748255
$l\sqrt{(\frac{9}{4}\,n\,n + 1)}$	=	2, 1440636	2, 1374128
$\sqrt{(\frac{9}{4}\,n\,n + 1)}$	=	139, 3361	137, 2185
add. $\frac{3}{2}\,n$	=	139, 3289	137, 2149

	278, 6650	274, 4334
$l(\frac{3}{2}n + \sqrt{(\frac{9}{4}nn + i)}) =$	2, 4450823	2, 4384369
l partis majoris $=$	0, 8150274	0, 8128123
l partis minoris $=$	9, 1849725	9, 1871876
Pars major $=$	6, 531718	6, 498489
Pars minor $=$	0, 153099	0, 153882
Eritque $\theta =$	6, 378619	6, 344607
$l\,\theta =$	0, 8047267	0, 8024047
$l\,\frac{2\delta}{b} =$	(−5), 8018185	(−5), 5440797
$l\,\frac{2\delta}{b}\,\theta =$	(−4), 6065452	(−4), 3464844
$2\,l\,\theta =$	1, 6094534	1, 6048094
$l\,\frac{2\delta}{b}\,\theta^3 =$	(−2), 2159986	(−3), 9512938
	0, 4771213	0, 4771213
$l\,\frac{2\delta}{3b}\,\theta^3 =$	(−3), 7388773	(−3), 4741725
A tang $\theta =$	81°,5′,24″	81°,2′,35″
seu	291924″	291755″
huj. log. $=$	5, 4652698	5, 4650183
add.	4, 6855749	4, 6855749
	0, 1508447	0, 1505932
$l\,\frac{2\delta}{b} =$	(−5), 8018185	(−5), 5440797
$l\,\frac{2\delta}{b}$ A tang $\theta =$	(−5), 9526632	(−5), 6946729
$\theta =$	6, 378619	6, 344607
subtr. $\frac{2\delta}{b}\,\theta =$	0, 000404	0, 000222

	6, 378215	6, 344385
add. $\frac{2\delta}{3b}\theta^3 =$	0, 005481	0, 002980
$\frac{2\delta}{b}$ A tang $\theta =$	0, 000089	0, 000049
$t =$ tang $\frac{1}{2} v =$	6, 383785	6, 347414
l tang $\frac{1}{2} v =$	10, 8050782	10, 8025969
$\frac{1}{2} v =$	81°, 5′,50″	81°,2′,49″
& $v =$	162, 11, 40″	162, 5, 38
$b =$	1357, 313	1371, 392
$a =$	678, 678	685, 708
$b - a =$	678, 635	685, 684
$l(b-a) =$	2, 8316362	2, 8361240
$la =$	2, 8316639	2, 8361393
$l\frac{b-a}{a} =$	9, 9999723	9, 9999847
$l -$ coſ $v =$	9, 9786825	9, 9784370
$l\frac{b-a}{a} =$	9, 9999723	9, 9999847
	9, 9786548	9, 9784217
$-\frac{b-a}{a}$ coſ $v =$	0, 9520392	0, 9515284
Denom. =	0, 0479607	0, 0484715
$lb = l$ num. =	3, 1326800	3, 1371615
l Den. =	8, 6808855	8, 6854865
$ly =$	4, 4517945	4, 4516750
Ab $v =$	162°,11′,40″	162°,5′,38″
ſubtr. diſt. Nodi a Perih. =	9, 13, 17	9, 8, 21
Diſt. Cometæ a Nodo =	152, 58, 23	152, 57,17

In triangulo ergo ſphærico ad B rectangulo CNP repræſentet N nodum aſcend. NP eclipticam & NC orbitam cometæ, dantur. Fig. 6.

 NC =

	N C =	152°,58′,23″	152°,57′,17″
	ang. CNP =	59, 32, 38	59, 46, 45
ſin CP = ſin NC. ſin N.	*l* ſin NC =	9, 6574473	9, 6577197
tang NP = coſ N. tang NC.	*l* ſin N =	9, 9355161	9, 9365599
	l ſin C P =	9, 5929634	9, 5942796
	Latitudo heliocentr C P =	23°,3′,39″	23°,8′,6″
	l coſ N =	9, 7049036	9, 7018564
	l — tang N C =	9, 7076705	9, 7080138
	l — tang N P =	9, 4125741	9, 4098702
	Ergo N P =	165°,30′,10″	165°,35′,20″
	ſeu N P =	5ˢ,15°,30′,10″	5ˢ,15°,35′,20″
	Add. longitudo Nodi =	9, 2, 31,32	9, 2, 24, 6
	Longitudo heliocentrica	2, 18, 1,42	2, 17,59,26
	Longitudo terræ =	3, 1,51,23	3, 1,51,23
Fig. 7.	Ergo ang. *c* S T =	13,49,41	13°,51,57
	C S *c* =	23, 3,39	23, 8, 6
	l y = *l* C S =	4, 4517945	4, 4516750
	add. { *l* ſin C S *c* =	9, 5929634	9, 5942796
	{ *l* coſ C S *c* =	9, 9638300	9, 9635903
	l C *c* =	4, 0447579	4, 0459546
	l S *c* =	4, 4156245	4, 4152653
	add. { *l* ſin *c* S T =	9, 3784143	9, 3795762
	{ *l* coſ *c* S T =	9, 9872269	9, 9871565
	l c P =	3, 7940388	3, 7948415
	l S P =	4, 4028514	4, 4024218
	S P =	25284, 32	25259, 32
	ab S T =	98275	98275
	Erit T P =	72990, 68	73015, 68
	A *l c* P =	3, 7940388	3, 7948415
	ſubtr. *l* T P =	4, 8632675	4, 8634162

l tang.

l tang. S T *c* =	8, 9307713	8, 9314253
S T *c* =	4°,52′,24″	4°,52′,51″
Addatur long. Solis	9 ˢ,1,51,23	9 ˢ,1,51,23
Long. Com. Geocentrica	9 ˢ, 6°,43′,47″	9 ˢ, 6°,44′,14″
A *l* T P =	4, 8632675	4, 8634162
subtr. *l* cos S T *c* =	9, 9984272	9, 9984223
l c T =	4, 8648403	4, 8649939
a l C *c* =	4, 0447579	4, 0459546
l tang C T *c* =	9, 1799176	9, 1809607
Latitudo Geocentrica	8°,36′,18″	8°,37′,32″

Accedit ergo prior hypothesis qua sumsimus $r = 72700$ propius ad veritatem, ex quo concludimus valorem verum ipsius r multo minorem esse debere assumto. Atque ex latitudinibus concluditur iste valor ipsius r diminui debere 835, ob longitudines autem diminui deberet 2763. Medium ergo sumendo foret valor ipsius $r = 72700 - 1800 = 70900$ Hac itaque positione orbita cometæ multo magis a parabola ad figuram ellipticam reducetur, qua cognita hujus cometæ tempus periodicum definiri poterit. Quæ investigatio cum operæ pretium sit, tribuamus iterum ipsi r duos valores intra quos verus contineatur.

Sit ergo r =	70000	72000
erit lr =	4, 8450980	4, 8573325
add. $l \sin \eta$ =	9, 6128990	9, 6128990
$l \cos \eta$ =	9, 9600122	9, 9600122
$l G \eta$ =	4, 4579970	4, 4702315
$l g \eta$ =	4, 8051102	4, 8173447

$g\eta =$

$g\eta =$	63842, 5	65666, 6
$g m =$	13539, 0	13539, 0
$g n =$	5875, 0	5875, 0
$m \eta =$	50303, 5	52127, 6
$n \eta =$	57967, 5	59791, 6
$Sg =$	98407, 0	98407, 0
$g\eta =$	63842, 5	65666, 6
$Sg + g\eta =$	162249, 5	164073, 6
$Sg - g\eta =$	34564, 5	32740, 4
$l\,(Sg - g\eta) =$	4, 5386302	4, 5150840
$l\,(Sg + g\eta) =$	5, 2101833	5, 2150387
	9, 3284469	9, 3000453
l tang $(90 - \frac{1}{2} Sg\eta) =$	10, 0897890	10, 0897890
l tang $\frac{1}{2}\,(g\eta S - g S\eta) =$	9, 4182359	9, 3898343
$\frac{1}{2}(g\eta S - g S\eta) =$	14°,40′,46″	13°,47′,12″
$\frac{1}{2}\,(g\eta S + g S\eta) =$	50, 52, 52	50, 52, 52
$g\eta S =$	65, 33, 38	64, 40, 4
$g S\eta =$	36, 12, 6	37, 5, 40
A $l Sg$ ſin $Sg\eta =$	14, 9838096	14, 9838096
ſubtr. l ſin $g\eta S =$	9, 9592327	9, 9560926
$l S\eta =$	5, 0245769	5, 0277170
a $l G\eta =$	4, 4579970	4, 4702315
l tang $GS\eta =$	9, 4334201	9, 4425145
$GS\eta =$	15°,10′,41″	15°,29′,2″
A $l G\eta =$	4, 4579970	4, 4702315
ſubtr. l ſin $GS\eta =$	9, 4180021	9, 4264582
$l SG =$	5, 0399949	5, 0437733
	12, 3790825	12, 3790825
ſubtr. 2 $l SG =$	10, 0799898	10, 0875466
$l GO =$	2, 2990927	2, 2915359
add l coſ $GS\eta =$	9, 9845798	9, 9839443

$l\eta o =$

$l\eta o =$	2, 2836725	2, 2754802
Ergo $\eta o =$	192, 164	188, 573
ab $S\eta =$	105822, 22	106590, 12
$So =$	105630, 06	106401, 55
Ang. $g\eta S =$	65°,33′,38″	64°,40′,4″
$\zeta m\eta =$	9, 54, 8	9, 54, 8
$\zeta lo =$	75, 27, 46	74, 34, 12
A $lm\eta =$	4, 7015982	4, 7170677
ſubtr. l ſin $\zeta lo =$	9, 9858686	9, 9840573
	4, 7157296	4, 7330104
add l ſin $\zeta m\eta =$	9, 2354458	9, 2354458
add l ſin $g\eta S =$	9, 9592327	9, 9560926
$ll\eta =$	3, 9511754	3, 9684562
$lml =$	4, 6749623	4, 6891030
Ergo $l\eta =$	8936, 67	9299, 43
ſubtr. $\eta o =$	192, 16	188, 57
$lo =$	8744, 51	9110, 86
$llo =$	3, 9417355	3, 9595593
add. $l\,\frac{\alpha+\beta}{\alpha} =$	0, 2991128	0, 2991128
$l\lambda\theta =$	4, 2408483	4, 2586721
Ab ang. $\zeta lo =$	75°,27′,46″	74°,34′,12″
ſubtr. $\zeta k\theta =$	17, 10, 24	17, 10, 24
$k\theta\lambda =$	58, 17, 22	57, 23, 48
A $l\lambda\theta =$	4, 2408483	4, 2586721
ſubtr. l ſin $\zeta k\theta =$	9, 4702096	9, 4702096
	4, 7706387	4, 7884625
add. l ſin $\zeta lo =$	9, 9858686	9, 9840573
add. l ſin $k\theta\lambda =$	9, 9297837	9, 9255293

$lk\theta =$	4, 7565073	4, 7725198
$lk\lambda =$	4, 7004224	4, 7139918
$ml =$	47311, 02	48876, 81
add. $km =$	3285, 17	3285, 17
$kl =$	50596, 19	52161, 98
ſubtr. $k\lambda =$	50167, 50	51759, 70
$l\lambda =$	428, 69	402, 28
$ll\lambda =$	2, 6321433	2, 6045284
add. $l\,\frac{\alpha}{\beta} =$	0, 0038429	0, 0038429
$ll\zeta =$	2, 6359862	2, 6083713
$kl =$	50596, 19	52161, 98
$l\zeta =$	432, 50	405, 85
$k\zeta =$	51028, 69	52567, 83
$k\theta =$	57082, 07	59227, 02
$k\theta + k\zeta =$	108110, 76	111794, 85
$k\theta - k\zeta =$	6053, 38	6659, 19
$l\,(k\theta - k\zeta) =$	3, 7819979	3, 8234215
$l\,(k\theta + k\zeta) =$	5, 0338690	5, 0484219
	8, 7481289	8, 7749996
l tang $(90 - \frac{1}{2}\zeta k\theta) =$	10, 8210294	10, 8210294
	9, 5691583	9, 5960290
$\frac{1}{2}\,(k\zeta\theta - k\theta\zeta) =$	20°,20′,44″	21°,31′,42″
$\frac{1}{2}\,(k\zeta\theta + k\theta\zeta) =$	81, 24 48	81, 24, 48
$k\zeta\theta =$	101, 45, 32	102, 56, 30
$k\theta\zeta =$	61, 4, 4	59, 53, 6
$\zeta lo =$	75°,27, 46	74, 34, 12
$k\zeta\theta + \zeta lo =$	177, 13, 18	177, 30, 42
$So\zeta =$	2, 46, 42	2, 29, 18
A l ſin $\zeta k\theta =$	9, 4702096	9, 4702096
ſubtr. l ſin $k\zeta\theta =$	9, 9907889	9, 9888258

add. $lk\theta$

=	9, 4794207	9, 4813838
add. $lk\theta$ =	4, 7565073	4, 7725198
$l\vartheta\zeta$ =	4, 2359280	4, 2539036
fm =	6672, 1	6672, 1
ml =	47311, 0	48876, 8
$l\zeta$ =	432, 5	405, 8
Erit $f\zeta$ =	54415, 6	55954, 8
hk =	17245, 1	17245, 1
$k\theta$ =	57082, 1	59227, 0
erit $h\theta$ =	74327, 2	76472, 1
$lf\zeta$ =	4, 7357234	4, 7478373
l tang ζ =	9, 6931129	9, 6931129
lF ζ =	4, 4288363	4, 4409502
$lh\theta$ =	4, 8711478	4, 8835030
l tang θ =	9, 6127775	9, 6127775
lH θ =	4, 4839253	4, 4962805
Ergo H θ =	30473, 7	31353, 1
F ζ =	26843, 3	27602, 6
H θ — F ζ =	3630, 4	3750, 5
l (H θ — F ζ) =	3, 5599545	3, 5740892
$l\zeta\theta$ =	4, 2359280	4, 2539036
l tang H N θ =	9, 3240265	9, 3201856
a lH θ =	4, 4839253	4, 4962805
lN θ =	5, 1598988	5, 1760949
Ang. H N θ =	11°,54′,28″	11°,48′,21″
N θ =	144510, 3	150001, 3
A $l\zeta\vartheta$ =	4, 2359280	4, 2539036
subtr. $l\,\frac{\alpha+\beta}{\beta}$ =	0, 3029557	0, 3029557
$l\vartheta o$ =	3, 9329723	3, 9509479
θo =	8569, 83	8931, 98

No =	135940, 5	141069, 3
$\frac{1}{2}\,So\zeta$ =	1°,23′,21″	1°,14′,39″
$90 - \frac{1}{2}\,So\zeta$ =	88, 36, 39	88, 45, 21
So =	105630, 06	106401, 55
No =	135940, 5	141069, 3
$No + So$ =	241570, 5	247470, 8
$No - So$ =	30310, 5	34667, 8
$l\,(No - So)$ =	4, 4815930	4, 5399263
$l\,(No + So)$ =	5, 3830439	5, 3935239
	9, 0985491	9, 1464024
l tang $(90 - \frac{1}{2}\,So\zeta)$ =	11, 6152831	11, 6631758
l tang $\frac{1}{2}\,(oSN - SNo)$ =	10, 7138322	10, 8095782
$\frac{1}{2}\,(oSN - SNo)$ =	79°, 3′,40″	81°,11′,15″
$\frac{1}{2}\,(oSN + SNo)$ =	88, 36, 39	88, 45, 21
oSN =	167, 40, 19	169, 56, 36
SNo =	9, 32, 59	7, 34, 6
ab l ſin $So\zeta$ =	8, 6854914	8, 6376495
ſubtr. l ſin oSN =	9, 3294159	9, 2420989
	9, 3560755	9, 3955506
$l\,No$ =	5, 1333489	5, 1494324
$l\,SN$ =	4, 4894244	4, 5449830
Ang. $gS\eta$ =	36, 12, 6	37, 5, 40
ſeu	1ˢ,6°, 12′,6″	1ˢ,7°, 5′,40″
oSN =	5, 17,40,19	5, 19,56,36
	6, 23,52,25	6, 27, 2,16
$a\ 6^s + g$ =	16, 0, 29, 2	16, 0, 29, 2
Long. Nodi aſcend.	9, 6, 36,37	9, 3, 26,46
l tang $HN\vartheta$ =	9, 3240265	9, 3201856
ſubtr. l ſin SNo =	9, 2198555	9, 1196138
	10, 1041710	10, 2005718

Inclinatio

Inclinatio Orb. ad Eclipt.	51°,48′,24″	57°,47′,2″
l cos SNo =	9, 9939394	9, 9962000
add. l cos HNϑ =	9, 9905525	9, 9907146
l cos SNH =	9, 9844919	9, 9869146
SNH =	15°,13′,15″	13°,59′,40″
a { l Fζ =	4, 4288363	4, 4409502
l Hϑ =	4, 4839253	4, 4962805
subtr. l sin HNϑ =	9, 3145790	9, 3109002
l FN =	5, 1142573	5, 1300500
l HN =	5, 1693463	5, 1853803
½ SNH =	7°,36′,37½″	6°,59′,50″
90 − ½ SNH =	82, 23, 22½	83, 0, 10″
SN =	30862, 02	35073, 82
NF =	130094, 03	134911, 87
NF+SN =	160956, 05	169985, 7
NF − SN =	99232, 01	99838, 05
l (NF − SN) =	4, 9966517	4, 9992961
l (NF+SN) =	5, 2067073	5, 2304124
	9, 7899444	9, 7688837
l tang (90−½SNH) =	10, 8741495	10, 9110303
l tang ½ (NSF−NFS) =	10, 6640939	10, 6799140
½ (NSF − NFS) =	77°,46′,18½″	78°,11′,48″
½ (NSF+NFS) =	82, 23, 22½	83, 0, 10
NSF =	160, 9, 41	161, 11, 58
NFS =	4, 37, 4	4, 48, 22
l sin SNH =	9, 4191955	9, 3835062
subt. l sin NFS =	8, 9058402	8, 9231624
	0, 5133553	0, 4603438
l SN =	4, 4894244	4, 5449830
l SF =	5, 0027797	5, 0053268

NH =	147688, 39	153242, 89
SN =	30862, 02	35073, 82
NH+SN =	178550, 41	188316, 71
NH − SN =	116826, 37	118169, 07
a l (NH − SN) =	5, 0675410	5, 0725038
l (NH+SN) =	5, 2517608	5, 2748889
	9, 8157802	9, 7976149
l tang (90−$\frac{1}{2}$SNH) =	10, 8741495	10, 9110303
	10, 6899297	10, 7086452
$\frac{1}{2}$ (NSH − NHS) =	78°,27′,30$\frac{2}{3}$″	78°,55′,59$\frac{1}{4}$
$\frac{1}{2}$ (NSH+NHS) =	82, 23, 22$\frac{1}{2}$	83, 0, 10
NSH =	160, 50, 53$\frac{1}{6}$	161, 56, 9 $\frac{1}{4}$
NSF =	160, 9, 41	161, 11, 58
FSH =	0, 41, 12$\frac{1}{6}$	0, 44, 11$\frac{1}{4}$
NHS =	3, 55, 52	4, 4, 11
A l sin SNH =	9, 4191955	9, 3835062
subtr. l sin NHS =	8, 8360518	8, 8510771
	0, 5831437	0, 5324291
lSN =	4, 4894244	4, 5449830
lSH =	5, 0725681	5, 0774121
FS = y =	100642, 1	101234, 1
HS = z =	118186, 6	119512, 2
Φ =	0°, 41′,12$\frac{1}{6}$	0, 44′,11$\frac{1}{4}$″
l sin Φ =	8, 0786123	8, 1090132
l cos Φ =	9, 9999688	9, 9999641
l cot Φ =	11, 9213565	11, 8909509
$l y$ =	5, 0027797	5, 0053268
$l z$ =	5, 0725681	5, 0774121
$l yz$ =	10, 0753478	10, 0827389

$l \eta\eta zz$ =

$l\,yyzz$ =	20, 1506956	20, 1654778
ſubtr.	13, 2823898	13, 2823898
	6, 8683058	6, 8830880
add $2\,l$ ſin Φ =	16, 1572246	16, 2180264
	3, 0255304	3, 1011144
Pars prior.	1060, 548	1262, 160
$l\,\sqrt{yz}$ =	5, 0376739	5, 0413694
	0, 4771213	0, 4771213
	4, 5605526	4, 5642481
$2\,l$ ſin Φ =	16, 1572246	16, 2180264
	0, 7177772	0, 7822745
Pars poſter. =	5, 221	6, 057
Ergo b =	1065, 769	1268, 217
y =	100642, 1	101234, 1
z =	118186, 6	119512, 2
$y - b$ =	99576, 4	99965, 9
$z - b$ =	117120, 9	118244, 0
A $l\,(z-b)$ =	5, 0686344	5, 0727790
ſubtr. $l\,(y-b)$ =	4, 9981564	4, 9998519
	0, 0704780	0, 0729271
add $l\,\frac{y}{z}$ =	9, 9302116	9, 9279147
	10, 0006896	10, 0008418
ſubtr. l ſin Φ =	8, 0786123	8, 1090132
	1, 9220773	1, 8918286
Pars ſubtr. =	83, 57518	77, 95224
a cot. Φ =	83, 43658	77, 79487
— tang v =	0, 13860	0, 15737
Anom. vera loci F = v =	172°, 6′,32″	171°, 3′,24″
ſubtr. FSN =	160, 9, 41	161°,11,58

Diſt. Nodi

Dist. Nodi ☊ a Perihel.	11, 56, 51	9, 51, 26
Ad $ly =$	5, 0027797	5, 0053268
add $l - \cos v =$	9, 9958679	9, 9946878
$l - y \cos v =$	4, 9986476	5, 0000146
add $lb =$	3, 0276634	3, 1031935
l Num.	8, 0263110	8, 1032081
$- y \cos v =$	99689, 10	100003, 36
$y - b =$	99576, 4	99965, 9
Den.	199265, 5	199969, 2
l Denom.	5, 2994320	5, 3009631
$la =$	2, 7268790	2, 8022450
Dist. Perih. a Sole $= a =$	533, 1864	634, 2274
Ergo $2a =$	1066, 3728	1268, 4548
$b =$	1065, 769	1268, 217
$\delta = 2a - b =$	0, 603	0, 237
$l\delta =$	9, 7803173	9, 3747483
$lb =$	3, 0276634	3, 1031935
Char. 10 minuenda. $ln =$	6, 7526539	6, 2715548
$\frac{1}{2} v =$	86°, 3′, 16″	85°, 31′, 42″
$lt =$	1, 1613272	1, 1067702
$lt^3 =$	3, 4839816	3, 3203106
$lt^5 =$	5, 8066360	5, 5338510
$lt^7 =$	8, 1292904	7, 7473914
$lnn =$	13, 5053078	12, 5431096
$lnt^5 =$	2, 5592899	1, 8054058
$ln^2 t^5 =$	9, 3119438	8, 0769606
$ln^2 t^7 =$	1, 6345982	0, 2905010
$ln^3 t^7 =$	8, 3872521	6, 5620558
$ln^3 t^9 =$	9, 5485793	7, 6688260

$t =$	14, 4986	12, 7870
$\frac{1}{3}t^3 =$	1015, 9220	696, 9303
$\frac{3}{5}n^2t^5 =$	0, 1231	71
$\frac{3}{7}n^2t^7 =$	18, 4766	8366
	1049, 0203	710, 5610
subtr. $\frac{2}{5}nt^5 =$	144, 9940	25, 5544
	904, 0263	685, 0066
subtr. $\frac{4}{7}n^3t^7 =$	139	2
$\frac{4}{9}n^3t^9 =$	1572	20
$t + \frac{1}{3}t^3 +$ &c.	903, 8552	685, 0044
$2\ la =$	5, 4537580	5, 6044900
$l\sqrt{b} =$	1, 5138317	1, 5515967
	3, 9399263	4, 0528933
$lm =$	5, 4345525	5, 4345525
	8, 5053738	8, 6183408
add. $l(t + \frac{1}{3}t^3 +$ &c.$) =$	2, 9560989	2, 8356934
	1, 4614727	1, 4540342
A Perih. ad F dies	28, 9383	28, 4468
seu consueto more	$28^d, 22^h, 31', 9''$	$28^d, 10^h, 43', 23''$
Com. in F. A. 1680. M. Dec.	36, 6, 1', 38	36, 6, 1, 38
Cometa per Perihelium transiit A. 1680. M. Dec. $=$	$7^d, 7^h, 30', 29''$	$7^d, 19^h, 18', 15''$
Dist. Perihelii a Sole $a =$	533, 1864	634, 2274
semilatus rectum $b =$	1065, 769	1268, 217
Dist. Nodi ☊ a Perihelio $=$	$11°, 56', 51''$	$9°, 51', 26''$
Long. Nodi ☊ helioc.	$9^s, 6°, 36', 37''$	$9^s, 3°, 26', 46''$
Inclinatio ad Eclipticam	$51°, 48', 24''$	$57°, 47', 2''$

Computetur nunc pari modo locus cometæ geocentricus.

ad A. 1680. M. Dec.	$12^d, 4^h, 46', 0''$	$12^d, 4^h, 46', 0''$
Tempus Perih. subtr.	7, 7, 30, 29	7, 18, 19, 15

$T =$	4, 21,15,30	4, 9,27,45
in diebus autem est $T =$	4, 8857	4, 3943
Ad Probl. VIII. $l\,(2a-b) =$	9, 7803173	9, 3747482
semiss.	9, 8901586	9, 6873742
$\frac{3}{2}\,l\,(2a-b) =$	9, 6704759	9, 0621225
$l\,T =$	0, 6889268	0, 6428897
add.	11, 0500076	11, 0500076
	11, 4094103	10, 7550198
subtr. $3\,la =$	8, 1806370	8, 4067350
$lu =$	3, 2287733	2, 3482848
Ergo $u =$	1693,''4	223,''0
seu Anomalia media $u =$	0°,28',13''	0°,3',43''
$b-a =$	532, 583	633, 990
$l\,(b-a) =$	2, 7263872	2, 8020824
$la =$	2, 7268790	2, 8022450
$l\,\frac{b-a}{a} =$	9, 9995082	9, 9998374
sit $\varrho =$	21°, 30'	10°, 30'
l sin $\varrho =$	9, 5640754	9, 2606330
add.	5, 3139333	5, 3142625
$l\,\frac{(b-a)}{a}$ sin $\varrho =$	4, 8780087	4, 5748955
$\frac{b-a}{a}$ sin $\varrho =$	75510,''7	37574,''7
add. $u =$	1693, 4	223, 0
	77204, 1	37797, 7
$u + \frac{b-a}{a}$ sin $\varrho =$	21°, 26',44''	10°, 29',57''
subtr. $\varrho =$	21°, 30', 0''	10°, 30', 0''

num. $=$	$-\,3', 16''$	$-\,3''$	
l cos $\varrho =$	9, 9686779	9, 9926661	
$l\,\dfrac{(b-a)}{a} =$	9, 9995082	9, 9998374	
	9, 9681861	9, 9925035	
$1-\dfrac{(b-a)}{a} =$	0, 070634	0, 017112	
erit $z =$	$-\,46',15''$	$-\,2', 55''$	
Erit $\omega =$	$20°, 43',5''$	$10°,27', 5''$	
$\frac{1}{2}\,\omega =$	$10°, 21',32\frac{1}{2}$	$5°, 13',32\frac{1}{2}''$	
l tang $\frac{1}{2}\,\omega =$	9, 2619649	8, 9612284	
subtr. $l\sqrt{\dfrac{2a-b}{b}} =$	8, 3763269	8, 1357774	
l tang $\frac{1}{2}\,v =$	10, 8856380	10, 8254510	
Ergo $\frac{1}{2}\,v =$	$82°, 35',9\frac{1}{2}''$	$81°,29',56\frac{1}{2}''$	
Et anomalia vera $v =$	165, 10, 19	$162,59',53$	
$l-$cos $v =$	9, 9852909	9, 9805920	
$l\,\dfrac{b-a}{a} =$	9, 9995082	9, 9998374	
	9, 9847991	9, 9804294	
$-\dfrac{(b-a)}{a}$ cos $v =$	0, 965604	0, 955937	
Den. $=$	0, 034395	0, 044062	
$l\,b =$	3, 0276634	3, 1031935	
lDen. $=$	8, 5364953	8, 6440642	
$l\,y =$	4, 4911681	4, 4591293	
Ab $v =$	$165°,10',10''$	$162°,59',53''$	
Dist. ☊ a Perih.	11, 56, 51	9, 51, 26	
N C $=$ Dist. Com. a ☊ $=$	$153°, 13',28''$	$153°, 8',27''$	Fig. 6.
CNP $=$	$51°, 48',24''$	57, 47, 2	

 $l$sin N C $=$

l ſin N C	=	9, 6536916	9, 6549454
l ſin N	=	9, 8953998	5, 9273925
l ſin C P	=	9, 5490914	9, 5823379
l coſ N	=	9, 7911844	9, 7268202
l—tang N C	=	9, 7029480	9, 7045221
l—tang N P	=	9, 4941324	9, 4313423
Lat. helioc. C P	=	20°,44′,12″	22°,28′,21″
Ergo N P	=	162, 40, 22	164, 53, 28
ſeu N P	=	5ˢ,12°, 40,22	5ˢ, 14,53,28
Add. long. ☊	=	9, 6, 36,37	9, 3,26, 46
Longitudo heliocentr.		2, 19,16,59	2ˢ, 18,20,14
Longitudo terræ		3, 1,51,23	3, 1,51,23
Ang. *c*S T	=	′12°,34,24″	13°,31′, 6″
C S *c*	=	20, 44, 12	22, 28, 21
ly=*l*C S	=	4, 4911681	4, 4591293
add. *l* ſin. C S *c*	=	9, 5490914	9, 5823379
add. *l* coſ C S *c*	=	9, 9709127	9, 9657017
l C *c*	=	4, 0402595	4, 0414672
l S *c*	=	4, 4620808	4, 4248310
add. { *l* ſin *c*S T	=	9, 3378364	9, 3687698
add. { *l* coſ *c*S T	=	9, 9894579	9, 9877967
*lc*P	=	3, 7999172	4, 7936208
*l*S P	=	4, 4515387	4, 4126277
S P	=	28283, 9	25859, 9
ab S T	=	98275,	98275, 0
T P	=	69991	72415,
a lc P	=	3, 7999172	3, 7936208
ſubtr. *l* T P	=	4, 8450422	4, 8598285
l tang S T *c*	=	8, 9548750	8, 9337923
S T *c*	=	5°,9′,1″	4°,54′,27″
Long. Solis		9ˢ,1, 51, 23	9, 1, 51, 23

Long.

Long. Com. Geocentr.	9, 7, 0, 24	9, 6, 45, 50
a l T P =	4, 8450422	4, 8598285
subtr. l. cos S T c =	9, 9982431	9, 9984053
$l\,c$ T =	4, 8467991	4, 8614232
a l C c =	4, 0402595	4, 0414672
l tang C T c =	9, 1934604	9, 1800440
Latitudo Geocentr. =	8°, 52′, 24″	8°, 36′, 27″

Hinc tam ex longitudinibus quam latitudinibus obſervatis liquet verum valorem ipſius r majorem eſſe quam 72000. Ex præcedente autem hypotheſi collegimus, r minorem eſſe quam 72700. Sunt autem omnes quatuor tam longitudines quam latitudines obſervatis majores, ex quo ſequitur, valores intra 72000 & 72700 contentos præbituros eſſe minores & longitudines & latitudines, ſicque veritati magis conſentaneas; ita ut intra hos limites minima longitudo & latitudo contineatur. Cum autem valores 72000 & 72700 pro r ſubſtituti æquales fere præbeant longitudines ac latitudines, minimum fere reſpondebit valori medio 72350; quem tanquam verum ipſius r valorem aſſumamus; cum eum propius definire vix liceat.

Comparemus ergo has duas hypotheſes inter ſe.

Hypotheſis r =	72000	72700
Cometa per perihelium tranſiit A. 1680 M Dec.	7^d, 19^h, 18′, 15″	7^d, 22^h, 17′, 48″
Diſt. Perih. a ſole — —	634, 227	678, 678
Semi latus rectum — —	1268, 217	1357, 313
Diſt. nodi ☊ a Perih. —	9°, 51′, 26	9°, 13′, 17″
Long. nodi ☊ helioc. —	9^s, 3°, 26, 46	9^s, 2°, 31′, 32″
Inclin. Orbitæ ad Eclipt.	57°, 47′, 2″	59°, 32′, 38″

 Sumendo

Sumendo ergo inter hæc ſyſtemata medium, prodibit Cometæ hujus ſequens theoria quaſi vera

I. Cometa per perihelium tranſiit A. 1680 M. Decembri	–	$7^d, 20^h, 48', 0''$
II. Diſtantia Perihelii a Sole	=	656, 4525
III. Semi - latus rectum	=	1312, 7650
IV. Diſtantia nodi ☊ a Perih.	=	$9°, 32', 21''$
V. Longitudo helioc. nodi ☊	=	$9^s, 2°, 59', 9'$
VI. Inclinatio Orbitæ ad Eclipt.	=	$58°, 39', 50''$

Non multum discrepant hæ orbitæ Cometæ determinationes ab iis, quas Neutonus & Hallejus invenerunt, quarum ille conſtructione geometrica, hic vero calculo eſt uſus, ita tamen ut locum nodorum, & inclinationem & tempus, quo per perihelium tranſiit, eadem retinuerit, quæ Neutonus per conſtructionem inveniſſet: uterque autem orbitam cometæ perfectam eſſe parabolam aſſumſit. Quare cum conſtructio geometrica in hoc negotio admodum ſit lubrica, atque Hallejus præcipua capita hinc nata retinuerit, mirum non eſt, quendam diſſenſum inter hanc orbitæ determinationem & Neutonianam deprehendi, neque enim eſſet mirandum, ſi iſte disſenſus major prodiiſſet. Statuit autem Neutonus I. hunc Cometam in perihelio eſſe verſatum A. 1680. M. Dec. $8^d, 0^h, 4'$; II. diſtantiam perihelii a ſole = 607, 5; III. ſemi-latus rectum = 1215. IV. Diſtantiam nodi a perihelio = $9°, 20'$: V. Long. nodi ☊ = $9^s, 1°, 53'$ & VI. inclinationem orbitæ ad Eclipt. $61°, 20', 20''$. Orbita autem hujus cometæ ſic proxime inventa poterit ſecundum

dum methodum, quam in ſuperiore diſſertatione de Cometa A. 1742. tradidi, ulterius corrigi, atque cum accuratiſſimæ obſervationes per ſatis longum temporis ſpatium ſuppetant, fere ad eum perfectionis gradum perduci, ut axis transverſus ejus orbitæ definiri, atque adeo ejus tempus revolutionis asſignari queat. Quem laborem aliis relinquens interim ex hac cometæ orbita inventa, tanquam eſſet vera, tempus periodicum inveſtigabo. Cum igitur ſit

Diſtantia perihelii a ſole	a	=	656, 4525
erit	$2a$	=	1312, 9050
ſubtrahatur	b	=	1312, 7650
erit	$2a - b$	=	0, 1400
At eſt	la	=	2, 8172032
unde	laa	=	5, 6344064
ſubtr.	$l(2a-b)$	=	9, 1461280
l ſemi-axis transverſi		=	6, 4882784
addatur ſemiſſis		=	3, 2441392
			9, 7324176
ſubtrahatur	$lc\sqrt{c}$	=	7, 5000000
			2, 2324176

Unde erit tempus periodicum = 170, 77 annorum. Reverteretur itaque iſte cometa ad perihelium poſt annos centenos & ſeptuaginta; & quamvis exiguum discrimen in valore $2a-b$ vehementer hoc tempus immutare poſſit, tamen non adeo a veritate aberrabit. Neque enim contra hanc reverſionem quicquam valet objectio, quod idem cometa annis 170 antea non ſit obſervatus, neque in Tabula Halleji unquam ante ullus ſit obſervatus, qui cum hoc conſentiret. Primum enim

ſtatus

ſtatus cometæ apparens plurimum a loco terræ in orbita ſua pendet, quæ ſi verſetur in iisdem ſignis, quibus cometa eſt proximus, cometa maxime fulgens conſpicietur, & caudæ longitudo apparens non ex ejus vera quantitate, ſed potiſſimum e ſitu relativo reſpectu noſtri æſtimatur. Utraque opportunitas locum habuit A. 1680, quo tempore non ſolum cometa ſatis
Fig. 8. prope ad locum terræ acceſſit, ſed etiam caudæ directio C c ita ratione terræ T erat diſpoſita, ut ejus magnitudo apparens ſeu angulus C T c, fuerit vehementer magnus; quanquam & hic cometa ob ſummam in perihelio vicinitatem cum ſole præ reliquis multo majori cauda inſtructus eſt putandus. Ex his autem intelligitur, ſi idem hic cometa vel in poſterum redeat, vel ante jam aliquoties ad perihelium acceſſerit, niſi terra eo tempore in iisdem fere orbitæ ſuæ locis ſit verſata, longe alia ſpecie hunc eundem cometam conſpici debuiſſe, ut ex terra viſus nullo modo pro eodem haberi potuerit. Tum vero ad quod maxime eſt attendendum, hic inſignis cometa ſæpius ad ſolem reverti poterit, incolis terræ prorſus inconſpicuus. In hoc enim Cometa idem, quod de cometa anni 1742 notavi, uſu venit, quod diſparuerit, fere antequam ejus diſtantia a terra diſtantiam ſolis ſuperaſſet. Quare cum terra eo tempore, quo iſte cometa ante redierit, in orbita ſua ita ſita eſſe poſſit, ut is perpetuo longius quam ſol a terra fuerit remotus, nequidem obſervari potuit; unde eo minus eſt mirandum, hunc cometam ante nunquam eſſe animadverſum, quantum quidem ex obſervationibus ſufficienti ſtudio inſtitutis colligere licet. Confirmatur

matur itaque maxime illa conjectura, quam ante feci, nullum cometam nobis esse conspicuum, nisi terræ sit propior quam sol, sicque sine dubio complures cometæ ad solem in orbitis suis sæpius revertuntur, aspectum nostrum prorsus fugientes. Et hanc ob rem numerus cometarum systema solare exornantium multo erit major, quam suspicamur; utrum autem quandoque cometas, qui ad aliarum stellarum fixarum systemata pertineant, spectare nobis liceat; hoc nullo modo affirmari poterit, cum eorum distantia plus quam millies superare deberet eam, in qua cometæ nostri systematis se demum spectandos exhibeant.

Quoniam ceterum cometæ sunt corpora non admodum ingentia, & terram in perigæo celerrime præterire solent, effectus eorum ratione attractionis in perturbatione motus terræ non minimus esse nequit: nisi forte proxime ad terram accedant; quod quidem fieri non potest, nisi dum in alterutro nodo versantes per ipsam terræ semitam transeant, ibique terram offendant; unde non solum ob attractionem mutuam, sed etiam a mutuo conflictu effectus maxime funesti resultare deberent. Quod autem in majoribus distantiis motum terræ vix perturbare valeant, ratio est, quia vis solis, qua terra in orbita sua retinetur, incomparabiliter est major, quam illa vis, quæ unquam a cometa proficisci posset; ita ut vis cometæ a vi solis longissime superetur ac penitus absorbeatur. Hoc vero tantum est tenendum de viribus cometarum, quarum directiones in planum eclipticæ incidunt; aliter enim comparata est ratio earum viri-

um, quarum directio ad planum eclipticæ eſt normalis. Cum enim hæ vires a vi ſolis non afficiantur, nihil impedit, quominus effectum ſuum producant; qui dum terram de plano eclipticæ vel boream vel auſtrum verſus retrahere nituntur, actu orbitam terræ in aliud planum deducent; unde cum axis terræ non afficiatur, obliquitas eclipticæ atque poſitio punctorum æquinoctialium & ſolſtitialium immutabitur, qui effectus eo magis erit ſenſibilis, quo propius cometa ad terram acceſſerit, ſimulque quo major eo tempore fuerit ejus latitudo. Hinc non dubitarim mutabilitatem obliquitatis eclipticæ, quæ nunc quidem extra dubium poſita videtur, pariter ac variationem, quam in ſtellarum fixarum longitudine & latitudine deprehendunt, ſoli cometarum actioni tanquam veræ cauſæ adſcribere; facile autem erit in poſterum hanc conjecturam per obſervationes vel confirmare vel refellere, cum ſi hypotheſis hæc eſſet vera, iſtæ mutationes ſubito poſt apparitionem cujusque cometæ contingere deberent. Quod quidem ad cometam anni 1742 attinet, ejus actione obliquitas eclipticæ aliquantum augeri debuiſſet, quamobrem Aſtronomi ſunt rogandi, ut quam primum fieri licet, obliquitatem eclipticæ nunc quidem omni ſtudio inveſtigent, ut intelligi poſſit, utrum ea major reperiatur, quam ante hujus cometæ apparitionem eſt inventa, an ſecus. Generatim vero cometarum, qui in perigæo ſatis prope ad terram accedunt, ſimulque notabilem latitudinem habent, effectus ita erit comparatus; ut ſequens tabella declarat:

Si cometæ Latitudo fuerit borealis &

Locus ſolis ſit in ♈. Puncta æquinoctialia non afficientur, obliquitas eclipticæ autem augebitur.

Locus ſolis ſit in ♋. Puncta æquinoctialia promovebuntur, obliquitas eclipticæ non mutabitur.

Locus ſolis ſit in ♎. Puncta æquinoctialia non afficientur, obliquitas eclipticæ diminuetur.

Locus ſolis in ♑. Puncta æquinoctialia regredientur, obliquitas eclipticæ non afficietur.

Sin cometæ latitudo fuerit auſtralis, effectus iſti erunt contrarii. Probe autem iſta æquinoctiorum variatio diſtingui debet a præceſſione æquinoctiorum ſatis perſpecta, quæ non a mutabilitate eclipticæ, ſed a mutatione ipſius axis terræ proficiſcitur, qui a cometis non variatur. Hinc autem recepta Aſtronomorum regula, qua ſtatuunt, puncta æquinoctialia quotannis 50″ regredi, non parum perturbabitur. Quod autem ab antiquiſſimis temporibus obliquitas eclipticæ ſatis ſenſibiliter ſit minuta, concludendum eſt, vel plures cometas cum latitudine boreali, dum ſol in ſignis borealibus verſabatur, vel cum latitudine auſtrali, dum ſol in ſignis auſtralibus verſabatur, ad terram acceſſiſſe; horumque effectum prævaluiſſe.

Investigatio
Orbitæ Cometæ
qui A. 1744. apparuit.

Observationes, ex quibus orbitam hujus cometæ ſum determinaturus, mecum Pariſiis ſunt communicatæ, quæ cum videantur omni cura inſtitutæ, perquam ſunt idoneæ, ad quas computus ſecundum methodum meam accommodetur. Prima quidem obſervatio facta eſt Lauſannæ jam die 13 Decembr. anni præterlapſi, quæ cum omnium, quas quidem adhuc mihi videre contigit, prima ſit, merito in hunc finem adhibetur. Hæc autem cum reliquis, quas accepi, ad meridianum Pariſinum revocata ſequenti modo ſe habebat.

Pariſiis Temp.appar.	Long.Cometæ	Lat.Com.bor.
1743 Dec.$13^d, 8^h, 1', 45''$	♈ $28^\circ, 26', 13''$	$15^\circ, 11', 0''$
1744 Jan. 3, 5, 27, 40	♈ 14, 11, 10	17, 32, 50
Jan. 7, 5, 1, 43	♈ 12, 3, 10	17, 51, 30
Jan. 18, 7, 2, 0	♈ 6, 57, 15	18, 37, 5

Quoniam igitur ad calculum inſtituendum mihi opus eſt tribus obſervationibus, inter quas temporum intervalla non nimis ſint inter ſe inæqualia, ex his elegi primam ſecundam & quartam, omittendo tertiam utpote ſecundæ nimis vicinam. Quia enim longitudo hujus cometæ hoc temporis intervallo parum fuit mutata, præſtat obſervationes aliquantum a ſe invicem remotas adhibere, quam nimis vicinas. Reductis ergo his obſervationibus ad tempus medium, ſupputatisque pro earum

mo-

momentis terræ ſolisve locis una cum diſtantiis ſolis a terra, ſequentia prodierunt data:

Ordo Obſ.	Berol. temp. medio	Longit. Comet.	Latitudo Com.
I.	1743 Dec. $13^d, 8^h, 40'$	$0^s, 28°, 26', 13''$	$15°, 11', 0''$
II.	1744 Jan. 3, 6, 17	0, 14, 11, 10	17, 32, 50
III.	Jan. 18, 7, 57	0, 6, 57, 15	18, 37, 5

Temp. Obſ.	Locus ſolis	Log. diſt. ☉ a Terra
I.	$8^s, 21°, 30', 14''$	4, 9 9 2 9 0 3
II.	9, 12, 48, 18	4, 9 9 2 7 2 1
III.	9, 28, 9, 37	4, 9 9 3 0 3 2

Temporis ergo intervalla inter primam & ſecundam itemque ſecundam & tertiam obſervationem erunt

Inter I & II.	$20^d, 21^h, 37'$
II & III.	15, 1, 40

Exprimantur horæ cum minutis in partibus diei decimalibus, ut obtineantur valores pro litteris α & β, quæ in calculum ingrediuntur, erit

$$\alpha = 20,9008 \quad | \quad l\,\alpha = 1,320136$$
$$\beta = 15,0694 \quad | \quad l\,\beta = 1,178096$$

Delineetur jam figura his obſervationibus conveniens, ſitque tabula planum eclipticæ repræſentante, S locus ſolis, f, g, h, loca terræ in ſua orbita momentis trium obſervationum; ſit porro $f\zeta$ longitudo cometæ in obſervatione prima, $g\eta$ in ſecunda & $h\theta$ in tertia. Ductisque his lineis, quarum prima & tertia ſe in k, prima autem & ſecunda ſe in m, & ſecunda cum tertia in q interſecet, erunt cum lineæ tum anguli: Fig. 9.

$l\,Sf = 4,\ 992903$	$Sf\zeta = 126^\circ,\ 55',\ 59''$
$l\,Sg = 4,\ 992721$	$Sg\eta = 91,\ 22,\ 52$
$l\,Sh = 4,\ 993032$	$Sh\theta = 68,\ 47,\ 38$
$fSg = 21^\circ, 18', 4''$	$fk\theta = 21,\ 28,\ 58$
$gSh = 15, 21, 19$	$fmg = 14,\ 15,\ 3$
$fSh = 36, 39, 28$	$g\eta h = 7,\ 13,\ 55$

His præparatis, ſi nota eſſet cometæ a terra diſtantia vera in obſervatione media, orbita cometæ poſſet determinari, deficiente autem hac notitia, plures diſtantiæ ſingi debebunt, ex ſingulisque orbita cometæ, quam eſſet habiturus, deduci, ut pateat, quænam hypotheſis proxime ad parabolam manuducat, quoniam tuto aſſumere poſſumus, veram cometæ orbitam parum a parabola diſcrepare. Siquis autem de hoc dubitet, poterit cometæ obſervatio quædam quarta ab aſſumtis ſatis remota in ſubſidium vocari, atque ex orbitis, quas ſingulæ hypotheſes ſuppeditaverunt, ad hoc tempus locus cometæ ſupputari, quo pateat, quænam earum proxime cum obſervatione hac quarta conveniat. In hunc finem adhibui obſervationem hic in Obſervatorio Academico factam die 18. Febr. cum cometa ſtellæ Marchab eſſet proximus, hinc autem collecta fuit.

Temp. medio Berol.	Longitudo Cometæ	Latitudo Bor.
A. 1744. Febr. $18^h, 6^h, 43''$	$11^s,\ 19^\circ,\ 57',\ 0''$	$19^\circ,\ 10',\ 56''$

Hoc autem tempore erat

Locus Solis $10^s, 29^\circ, 30', 40''$

& log. diſt. ſolis a terra $= 4,\ 995309$.

Pluribus igitur factis hypotheſibus circa diſtantiam cometæ a terra

terra in obſervatione media, eam maxime huic obſervationi ſatisfacere deprehendi, quæ ſimul proxime parabolam exhiberet. Primum quidem ſuſpicatus ſum, cum iſte cometa tantopere fulgeret, eum a nobis non admodum fuiſſe remotum, ideoque initio hanc diſtantiam finxi, 20000 & 30000 poſita media terræ a ſole diſtantia 100000, hinc autem orbita prodiit elliptica parum excentrica, non multum a circulo discrepans; unde hæ hypotheſes nimium a veritate abhorrebant. Majores igitur valores tribui huic diſtantiæ, neque prius orbita in hyperbolam abire coeperat, quam iſtam diſtantiam = 110000 poſuiſſem; limites autem, intra quos ea diſtantiæ magnitudo, quæ obſervationi quartæ ſatisfaceret, inveni 101000 & 106000; ſicque præter expectationem diſtantia cometæ a terra multo major evaſit, quam initio putaveram. Cometa igitur a nobis fere æque erat remotus ac ſol, & cum ejus diameter apparens æſtimaretur unius circiter minuti primi, ejus diameter vera ad diametrum terræ rationem fere habebat triplam.

Sit igitur G verus cometæ locus in obſervatione ſecunda, ex quo ad eclipticam demittatur perpendiculum $G\eta$, & quoniam diſtantia Gg tanquam cognita aſſumitur, ob angulum $Gg\eta$ latitudini obſervatæ, 17° 32′, 50″ æqualem; prodibit $G\eta = Gg.$ ſin $Gg\eta$ & $g\eta = Gg.\cos Gg\eta$. Factis ergo binis memoratis hypotheſibus, calculus ſequenti modo inſtituatur:

Hypoth.

	Hypoth. A.	Hypoth. B.
$Gg =$	101000	106000
$l\,Gg =$	5, 004321	5, 025206
add. $\left\{\begin{array}{l} l \text{ sin } Gg\eta = \\ l \text{ cos. } Gg\eta = \end{array}\right.$	9, 479275 9, 979306	9, 479275 9, 979306
$l\,G\eta =$	4, 483596	4, 504581
$l\,g\eta =$	4, 983627	5, 004612

Ducatur nunc ex sole recta $S\eta$, & cum in triangulo $Sg\eta$ dentur latera Sg, $g\eta$ cum angulo intercepto $Sg\eta$ $= 91°, 22°, 52,''$ erit angulorum reliquorum summa $= 88, 37, 8,$ & semisumma $= 44°, 18', 34''$ unde per trigonometriam reliqui anguli reperiuntur, quibus inventis erit $S\eta = \frac{Sg.\,\text{sin}\, Sg\eta}{\text{sin}\, S\eta g}$.

	A.	B.
A $l\,Sg =$	4, 992721	4, 992721
subtr. $l\,g\eta =$	4, 983627	5, 004612
l tang.	10, 009094	− 10, 011891
anguli.	45°,36',−½''	−45°,47'3''.6
subtrahatur	45°	45°
rest. ang.	0, 36,−½	0, 47, 3. 6
l tang. =	8, 019943	8, 136401
l tang. ½ summæ.	9, 989530	9, 989530
l. tang. ½ diff.	8, 009473	8, 125931
½ diff. =	0, 35, 8	0, 45, 56
½ summa =	44, 18, 34	44, 18, 34
$S\eta g =$	44, 53, 42	43, 32, 38
$gS\eta =$	43, 43, 26	45, 4, 30

Porro

	A	B
Porro est $lSg =$	4, 992721	4, 992721
l sin $Sg\eta =$	9, 999874	9, 999874
	4, 992595	4, 992595
subtr. l sin $S\eta g =$	9, 848687	9, 838162
$lS\eta =$	5, 143908	5, 154433
& $S\eta =$	139287	142703

Quia nunc in triangulo $GS\eta$ ad η rectangulo dantur latera $S\eta$ & $G\eta$, erit tang $GS\eta = \frac{G\eta}{S\eta}$ & $GS = \frac{S\eta}{\text{cos}\, GS\eta}$.

	A	B
A $lG\eta =$	4, 483596	4, 504581
subtr. $lS\eta =$	5, 143908	5, 154433
l tang $GS\eta =$	9, 339688	9, 350148
II. Lat. hel. $GS\eta =$	12, 19,55	12, 37,23
subtr. l cos $GS\eta =$	9, 989861	9, 989374
a $lS\eta =$	5, 143908	5, 154433
II. Dist. Com. a ⊙ $lSG =$	5, 154047	5, 165059

Invento puncto G, sint F & H loca cometæ vera in prima ac tertia observatione, & ducta corda FH secet SG in O. Ostendi autem in dissertatione mea fore intervallum $GO = \frac{2c^3 \,\text{sin}\, \alpha\tau . \,\text{sin}\, \beta\tau}{SG^2 \,\text{cos}(\alpha - \beta)\tau}$, ubi τ denotat motum terræ semidiurnum medium 29′, 34″. 098, ita ut in minutis secundis sit $\tau = 1774, 098$, & $l\tau = 3, 248977$, hinc ob litterarum α & β valores ante datos, reperiuntur anguli $\alpha\tau$ & $\beta\tau$ hoc modo:

$l\tau =$	3, 248977	
add. $\begin{cases} l\alpha = \\ l\beta = \end{cases}$	1, 320163 1, 178096	
$l\alpha\tau =$	4, 569140	
$l\beta\tau =$	4, 427073	
unde $\alpha\tau =$	37080″ =	10°,18′,0″
$\beta\tau =$	26734 =	7, 25, 34
$(\alpha - \beta)\tau =$	10346 =	2, 52, 26

Cum igitur c denotet diſtantiam ſolis a terra mediam = 100000, valor ſagittæ GO ſequenti modo definietur.

	A	B
add $\begin{cases} l\,\text{ſin}\ \alpha\tau = \\ l\,\text{ſin}\ \beta\tau = \end{cases}$	9, 252373 9, 111422	
	8, 363795	
ſubtr. l coſ $(\alpha - \beta)\tau =$	9, 999453	
	8, 364342	
add. $l\,2c^3 =$	15, 301030	
	13, 665372	13, 665372
ſubtr. 2 lSG =	10, 308094	10, 330118
lGO =	3, 357278	3, 335254
add. l coſ GSη =	9, 989861	9, 989374
prodibit $l\eta o$ =	3, 347139	3, 324628
unde ηo =	2224	2112
ſubtr. ab Sη =	139287	142703
remanebit So =	137063	140591

demiſſo ſcilicet ex puncto O in planum eclipticæ perpendiculo Oo. Nunc ſecet recta Sη reliquas cometæ longitudines in μ & ν, ad quæ puncta invenienda in triangulo S$f\mu$ primum dantur:

	A	B
$lSf =$	4, 992903	4, 992903
ang. $Sf\mu =$	126, 55, 59	126, 55, 59
cujus deinc.	53, 4, 1	53, 4, 1
Ob ang. $gS\eta =$	43, 43, 26	45, 4, 30
ſubtr. $fSg =$	21, 18, 4	21, 18, 4
remanebit $fS\mu =$	22, 25, 22	23, 46, 26
qui ablatus ab $Sfk =$	53, 4, 1	53, 4, 1
relinquet $S\mu f =$	30, 38, 39	29, 17, 35

Ob datos ergo omnes angulos erit $f\mu = \frac{Sf.\ \text{ſin}\ fS\mu}{\text{ſin}\ S\mu f}$ & $S\mu = \frac{Sf.\ \text{ſin}\ Sf\mu}{\text{ſin}\ S\mu f}$, unde calculus dabit

A $lSf =$	4, 992903	4, 992903
ſubtr. l ſin $S\mu f =$	9, 707318	9, 689554
	5, 285585	5, 303349
add. l ſin $fS\mu =$	9, 581424	9, 605443
add. l ſin $Sf\mu =$	9, 902730	9, 902730
$lf\mu =$	4, 867009	4, 908792
$lS\mu =$	5, 188315	5, 206079
Ergo $f\mu =$	73622	81057
$S\mu =$	154282	160723
ſubtr. $So =$	137063	140591
reſtabit $o\mu =$	17219	20132

ſimili modo in triangulo Sbv data reperiuntur.

$lSb =$	4, 993032	4, 993032
ang. $Sbv =$	68, 47,38	68, 47,38
Ejus deinceps:	111, 12,22	111, 12,22
Deinde ob $gS\eta =$	43, 43,26	45, 4,30
add. $gSb =$	15, 21,19	15, 21,19

 erit

		A.	B
erit bSv	=	59, 4, 45	60, 25, 49
qui ablatus ab externo	=	111, 12, 22	111, 12, 22
relinquit Svb	=	52, 7, 37	50, 46, 33

Ob datos ergo omnes angulos cum latere Sb erit $bv = \frac{Sb.\,\text{fin}\,bSv}{\text{fin}\,Svb}$ & $Sv = \frac{Sb.\,\text{fin}\,Sbv}{\text{fin}\,Svb}$. Ergo

		A	B
A lSb	=	4, 993032	4, 993032
fubtr. l fin Svb	=	9, 897282	9, 889121
		5, 095750	5, 103911
add. { l fin bSv	=	9, 933425	9, 939397
add. { l fin Sbv	=	9, 969548	9, 969548
lbv	=	5, 029175	5, 043308
lSv	=	5, 065298	5, 073459
Ergo bv	=	106949	110486
Sv	=	116225	118429
fubtr. ab So	=	137063	140591
remanebit ov	=	20838	22162

Nunc per punctum o duci debet linea recta $\zeta o \theta$, cujus partes ζo & θo fint in ratione temporum $\alpha : \beta$. Producatur ergo ov usque ad i, ut fit $oi : vo = \alpha : \beta$. feu $oi = \frac{\alpha}{\beta}.\,ov$, tum ducatur recta $i\zeta$ parallela ipfi bv, eritque recta $\zeta o \theta$ recta quæfita.

		A	B
Ad lov	=	4, 318856	4, 345609
add. $l\alpha : \beta$	=	0, 142067	0, 142067
loi	=	4, 460923	4, 487676
Ergo oi	=	28902	30738
fubtr. $o\mu$	=	17219	20132
remanebit μi	=	11683	10606

Nunc

Nunc in triangulo $\mu\zeta i$ dantur omnes anguli cum latere μi

	A	B
$l\mu i =$	4, 067565	4, 025551
ang. $\zeta\mu i =$	30, 38, 39	29, 17, 35
$\mu\zeta i =$	21, 28, 58	21, 28, 58
$180 - \mu i\zeta =$	52, 7, 37	50, 46, 33

Erit ergo $\zeta\mu = \dfrac{\mu i.\ \text{fin}\ \mu i\zeta}{\text{fin}\ \mu\zeta i}$ & $\zeta i = \dfrac{\mu i.\ \text{fin}\ \zeta\mu i}{\text{fin}\ \mu\zeta i}$.

	A	B
A $l\mu i =$	4, 067565	4, 025551
fubtr. l fin $\mu\zeta i =$	9, 563743	9, 563743
	4, 503822	4, 461808
add. l fin $\mu i\zeta =$	9, 897282	9, 889121
add. l fin $\zeta\mu i =$	9, 707318	9, 689554
$l\zeta\mu =$	4, 401104	4, 350929
$l\zeta i =$	4, 211140	4, 151362
Ergo $\zeta\mu =$	25183	22435
add. $f\mu =$	73622	81057
erit $f\zeta =$	98805	103492

Jam in triangulo $oi\zeta$ dantur duo latera cum angulo intercepto $oi\zeta$

	A	B
A $loi =$	4, 460923	4, 487676
fubtr. $l\zeta i =$	4, 211140	4, 151362
l tang:	10, 249783	10, 336314
ang:	60, 38,13	65,15,3.8
fubtr.	45	45
	15, 38,13	20, 15,3.8
fumma ang:	52, 7,37	50, 46,33
femi fumma.	26, 3,48.5	25, 23,16.5
l tang. femif:	9, 689401	9, 676306
l tang. ang.	9, 447002	9, 566955

	A	B
l tang semid:	9, 136403	9, 243261
semi diff.	7, 47, 43	9, 55, 52
semi summa	26, 3, 48	25, 23, 16
$o\zeta i =$	33, 51, 31	35, 19, 8
$\zeta o i =$	18, 16, 5	15, 27, 24

Porro est $o\zeta = \frac{oi.\ \text{sin}\ oi\zeta}{\text{sin}\ oi\zeta}$ unde invenitur hoc modo

Ad $loi =$	4, 460923	4, 487676
add. l sin $oi\zeta =$	9, 897282	9, 889121
	4, 358205	4, 376797
subtr. l sin $o\zeta i =$	9, 745968	9, 762022
$lo\zeta =$	4, 612237	4, 614775

si itaque corda $\theta\zeta$ producatur in n ob $b\theta n = o\zeta i$ erit

ang. $b\theta n =$	33, 51, 31	35, 19, 8

Nunc ob triangula $vo\theta$ & $io\zeta$ similia erit $\theta v = \frac{\beta}{\alpha}.\ \zeta i$ &

$\theta o = \frac{\beta}{\alpha}.\ o\zeta$, unde sequenti modo invenientur

A $l\zeta i =$	4, 211140	4, 151362
subtr. $l\alpha : \beta =$	0, 142067	0, 142067
$l\theta v =$	4, 069073	4, 009295
$\theta v =$	11724	10216
subtr. a $bv =$	106949	110486
erit $b\theta =$	95225	100270
A $lo\zeta =$	4, 612237	4, 614775
subtr. $l\alpha : \beta$	0, 142067	0, 142067
$l\theta o =$	4, 470170	4, 472708
At est $o\zeta =$	40948	41188
& $o\theta =$	29524	29696
unde $\zeta\theta =$	70472	70884

Inventis punctis ζ & θ ducantur rectæ $S\zeta$ & $S\theta$, ad quas inveniendas consideretur primum triangulum $Sf\zeta$, in quo ob data latera Sf & $f\zeta$ cum angulo intercepto $Sf\zeta$ inveniantur reliqua hoc modo.

subtr. $lSf =$	4, 992903	4, 992903
ab $lf\zeta =$	4, 994779	5, 014906
l tang:	10, 001876	10, 022003
ang:	45, 7,25.5	46, 27,2.8
	45,	45,
	0, 7,25.5	1, 27,2.8
summa angul.	53,4, 1	53, 4,1
semi summa	26,32,0.5	26,32,0.5
l tang. semis.	9, 698371	9, 698371
l tang. ang.	7, 334519	8, 403571
l tang semid:	7, 032890	8, 101942
semi diff:	0, 3, 42	0,43, 28
semi summa	26, 32, 0	26, 32, 0
$fS\zeta =$	26, 35, 42	27, 15, 28
$S\zeta f =$	26, 28, 18	25, 48, 32

Jam erit $S\zeta = \frac{Sf.\ \sin Sf\zeta}{\sin S\zeta f}$, hinc fiet

$lSf =$	4, 992903	4, 992903
add. $l\, Sf\zeta =$	9, 902730	9, 902730
	4, 895633	4, 895633
subtr. l sin $S\zeta f =$	9, 649096	9, 638859
$lS\zeta =$	5, 246537	5, 256774

Simili modo in triangulo $Sb\theta$ ob data latera Sb, $b\theta$ cum angulo intercepto invenietur:

A $lSb =$

	A	B
A lSb =	4, 993032	4, 993032
ſubtr. $lb\theta$ =	4, 978751	5, 001171
l tang:	10, 014281	−10,008139
anguli =	45,56,30.7	45, 32,12.6
ſubtr. =	45	45
	0, 46,30.7	0, 32,12.6
ſumma ang:	111, 12,22	111, 12,22
ſemi ſumma =	55, 36, 11	55, 36, 11
l tang ſemi ſum: =	10, 164540	10, 164540
l tang. ang: =	8, 131313	7, 971728
l tang. ſemid: =	8, 295853	8, 136268
ſemidiff. =	1, 7, 56	0, 47, 3
ſemi ſumma =	55, 36, 11	55, 36, 11
$bS\theta$ =	54, 28, 15	56, 23, 14
$S\theta b$ =	56, 44, 7	54, 49, 8

Deinde cum ſit $S\theta = \dfrac{Sb.\ \text{ſin}\ Sb\theta}{\text{ſin}\ S\theta b}$ fiet

Ad lSb =	4, 993032	4, 993032
add. l ſin $Sb\theta$ =	9, 969548	9, 969548
	4, 962580	4, 962580
ſubtr. l ſin $S\theta b$ =	9, 922281	9, 912399
$lS\theta$ =	5, 040299	5, 050181

Ex his longitudo Cometæ heliocentrica ad tempora trium obſervationum determinabitur hoc modo.

Cum ſit $fS\zeta$ =	26, 35, 42	27, 15, 28
Long. terræ ad I.	2, 21,30,14	2, 21, 30,14
I. Long. Cometæ helioc.	1, 24,54,32	1, 24, 14,46
ſubtr. $gS\eta$ =	1, 13,43,26	1, 15, 4,30
a Long. terræ ad II.	3, 12,48,18	3, 12,48,18

II. Long.

	A	B
II. Long. Cometæ helioc.	1, 29, 4,52	1, 27,43,48
Porro $bS\vartheta$ =	1, 24,28,15	1, 26,23,14
Long. terræ ad III	3, 28, 9,37	3, 28, 9,37
III. Long. Cometæ helioc. =	2, 3,41,22	2, 1,46,23
Hinc erit ang. $\zeta S\vartheta$ =	8,46,50	7,31,37

Cum igitur longitudo cometæ a prima ad tertiam obſervationem creſcat, manifeſtum eſt, verum cometæ motum fuiſſe directum, etiamſi ex terra retrogradus apparuerit.

Deinde etiam poſitio rectæ $\zeta\theta$ reſpectu linearum $S\zeta$ & $S\theta$ cognoſcetur, hac linea ultra θ in n producenda

Cum enim ſit $b\vartheta n$ =	33, 51, 31	35, 19, 8
ſubtr. ab $S\vartheta b$ =	56, 44, 7	54, 49, 8
erit $S\vartheta n$ =	22, 52, 36	19, 30, 0
ſubtr. $\vartheta S\zeta$ =	8, 46, 50	7, 31, 37
prodit ang. $S\zeta n$ =	14, 5, 46	11, 58, 23

Ex latitudinibus nunc geocentricis, inveniuntur perpendicula $F\zeta$ & $H\theta$, cum veris cometæ a terra diſtantiis Ff, Hb: Erit enim $F\zeta = f\zeta \cdot$ tang lat. I. $Ff = \frac{f\zeta}{\text{coſ. lat. I.}}$: atque $H\theta = b\theta$ tang lat. III. & $Hb = \frac{b\theta}{\text{coſ. lat. III.}}$.

$lf\zeta$ =	3, 994779	5, 014906
add. l tang. lat. I	9, 433580	9, 433580
ſubtr. l coſ lat. I	9, 984569	9, 984569
erit $lF\zeta$ =	4, 428359	4, 448486
& lFf =	5, 010210	5, 030337
Hinc ſit $F\zeta$ =	26813	28085
I. Diſt. Com. a terra Ff =	102379	107235

	A	B
Porro $l b\vartheta$ =	4, 978751	5, 001171
add. l tang lat. III =	9, 527485	9, 527485
subtr. l cos lat. III =	9, 976656	9, 976656
erit $l H\vartheta$ =	4, 506236	4, 529656
& $l H b$ =	5, 002095	5, 024515
Hinc fit $H\vartheta$ =	32080	33780
III. Dist. Com. a terra $H b$ =	100484	105807
II. Dist. Com. a terra $G g$ =	101000	106000

Cognitis nunc perpendiculis $H\vartheta$ & $F\zeta$, producatur recta HF, donec cum $\vartheta\zeta$ prolongata concurrat in N, eritque recta SN linea nodorum cometæ: erit autem $\frac{H\vartheta - F\zeta}{\zeta\vartheta}$ = tang $HN\vartheta$ & $\vartheta N = \frac{H\vartheta}{\text{tang } HN\vartheta}$: Ergo

	A	B
A $H\vartheta$ =	32080	33780
subtr. $F\zeta$ =	26813	28085
erit $H\vartheta - F\zeta$ =	5267	5695
& $l(H\vartheta - F\zeta)$ =	3, 721563	3, 755494
subtr. $l\zeta\vartheta$ =	4, 848017	4, 850548
l tang $HN\vartheta$ =	8, 873546	8, 904946
subtr. a $l H\vartheta$ =	4, 506236	4, 528656
$l\vartheta N$ =	5, 632690	5, 623710

Consideretur nunc triangulum $S\vartheta N$, in quo cum dentur latera $S\vartheta$. ϑN cum angulo intercepto, reperietur angulus ϑSN.

A $l\vartheta N$ =	4, 632690	5, 623710
subtr. $l S\vartheta$ =	5, 040299	5, 050181

l tang.

	A	B
l tang.	10, 592391	10, 573529
ang.	75, 39, 39	75, 3, 7
ſubtr.	45	45
	30, 39, 39	30, 3, 7
ſumma ang: $S\vartheta n$ =	22, 52, 36	19, 30, 0
ſemi ſumma =	11, 26, 18	9, 45, 0
l tang ſemiſ: =	9, 306063	9, 235102
l tang ang: =	9, 772930	9, 762348
l tang ſemid: =	9, 078993	8, 997450
Ergo ſemid:	6, 50, 23	5, 40, 38
ſemiſ:	11, 26, 18	9, 45, 0
Ang. ϑSN =	18, 16, 41	15, 25, 38
III. Long. helioc. puncti ϑ =	2, 3, 41,22	2, 1, 46,23
Longit. Nodi Aſcend: ☊ =	1, 15,24,41	1, 16,20,45

Demittatur nunc ex ϑ in lineam nodorum SN perpendiculum ϑP, ductaque HP erit angulus $HP\vartheta$ æqualis inclinationi orbitæ cometæ ad Eclipticam; Fiet autem $\vartheta P = S\vartheta$ ſin ϑSN & tang. $HP\vartheta = \frac{H\vartheta}{\vartheta P}$.

	A	B
Ad $lS\vartheta$ =	5, 040299	5, 050181
add. *l* ſin ϑSN =	9, 496415	9, 424905
$l\vartheta P$ =	4, 536714	4, 475086
a $lH\vartheta$ =	4, 506236	4, 528656
l tang $HP\vartheta$ =	9, 969522	10, 053570
Ergo ang. $HP\vartheta$ = =	42, 59,28	48, 31,29
ideoque Inclinatio Orbitæ Cometæ ad Eclipticam =	42, 59, 28	48, 31, 29

Determinemus nunc quoque latitudines cometæ heliocentricas; quæ erunt: tang. $FS\zeta = \frac{F\zeta}{S\zeta}$ & tang $HS\vartheta = \frac{H\vartheta}{S\vartheta}$. Diſtantiæ vero cometæ a Sole erunt $SF = \frac{S\zeta}{\cos FS\zeta}$ & $SH = \frac{S\vartheta}{\cos HS\vartheta}$.

A $lF\zeta$ =	4, 428359	4, 448486
ſubtr. $lS\zeta$ =	5, 246537	5, 256774
l tang. $FS\zeta$ =	9, 181822	9, 191712
I. Lat. helioc. $FS\zeta$ =	8, 38, 32	8, 50, 18
A $lS\zeta$ =	5, 246537	5, 256774
ſubtr. l coſ $FS\zeta$ =	9, 995040	9, 994813
I. Diſt. Com. a Sole lSF =	5, 251497	5, 261961
A $lH\vartheta$ =	4, 506236	4, 528656
ſubtr. $lS\vartheta$ =	5, 040299	5, 050181
l tang $HS\vartheta$ =	9, 465937	9, 478475
III. Lat. helioc. $HS\vartheta$ =	16, 17,51	16, 44,54
A $lS\vartheta$ =	5, 040299	5, 050181
ſubtr. l coſ $HS\vartheta$ =	9, 982188	9, 981175
III. Diſt. Com. a Sole. lSH =	5, 058111	5, 069006

Determinemus nunc quoque elongationes cometæ heliocentricas a nodo aſcendente ſeu linea SN, eritque $\cos FSN = \cos FS\zeta . \cos \zeta SN$ & $\cos HSN = \cos HS\vartheta . \cos \vartheta SN$.

A ϑSN =	18,16,41	15,25,38
ſubtr. $\vartheta S\zeta$ =	8,46,50	7,31,37
remanet ζSN =	9,29,51	7,54, 1
l coſ $FS\zeta$ =	9, 995040	9, 994813
l coſ ζSN =	9, 994006	9, 995858

	A	B
l cos FSN =	9, 989046	9, 990671
Ergo ang. FSN =	12, 48, 52	11, 49, 59
l cos HS♃ =	9, 982188	9, 981175
l cos θSN =	9, 977516	9, 984063
l cos HSN =	9, 959704	9, 965238
Ergo ang. HSN =	24, 18, 6	22, 37, 11
subtr. FSN =	12, 48, 52	11, 49, 59
Erit ang. FSH =	11, 29, 14	10, 47, 12

Quoniam igitur duo habemus puncta cometæ in orbita vera, nempe F & H, quorum distantiæ a foco S una cum angulo FSH sunt cognitæ, hinc naturam orbitæ determinare poterimus. Quia vero SH < SF apparet cometam his observationum temporibus ad suum Perihelium accessisse. Sit igitur Fig. 10. AHF vera orbita cometæ, circa focum S, quem sol occupat, descripta, cujus vertex seu perihelium sit in A. Sit distantia perihelii a sole AS $= a$. applicata ex foco S ad axem normalis seu semilatus rectum SB $= b$, anomalia vera seu angulus ASH $= v$; tum vero ponantur cognita SH $= y$; SF $= z$; & angulus FSH $= \phi$. atque tempus, quo cometa spatium FH percurrit, in diebus expressum sit $= T$. Ex his primum invenitur semilatus rectum $b = \frac{y^2 z^2}{4m^2 T^2} (\sin\phi)^2 + \frac{\sqrt{yz}}{3} (\sin\phi)^2$ existente $m = 271989,735$ & $l\,m = 5,4345525$. hincque $l\,2m = 5,7355825$. Deinde fit tang $v = \cot\phi - \frac{(z-b)y}{(y-b)z\sin\phi}$.

tandemque $a = \frac{by\cos v}{b-y+y\cos v}$. Ponatur postea $\frac{2a-b}{b} = n$;

 tangens-

tangensque anguli $\frac{1}{2}\,v = t$ fietque, fi orbita proxime ad parabolam accedat, tempus quo cometa a loco H in perihelium A pervenit

$$\frac{aa}{m\sqrt{b}}\left(\begin{array}{l} t + \frac{1}{3}t^3 - \frac{2}{5}nt^5 + \frac{3}{7}n^2t^7 - \frac{4}{9}n^3t^9 + \&c. \\ \qquad + \frac{3}{5}n^2t^5 - \frac{4}{7}n^3t^7 + \frac{5}{9}n^4t^9 - \&c. \end{array}\right)$$

quod tempus erit expreffum in diebus, dieique partibus decimalibus. Quia igitur tempus tertiæ obfervationis, quo cometa in puncto H hæfit, cognitum eft, hinc momentum, quo per perihelium tranfiit cognofcitur.

		A.	B.
Erit ergo $l\,y$	=	5, 058111	5, 069006
$l\,z$	=	5, 251497	5, 261961
$l\,y\,z$	=	10, 309608	10, 330967
$T = \alpha + \beta$	=	35, 9702	
$l\,T$	=	1, 555941	
ϕ	=	11, 29, 14	10, 47, 12
l fin ϕ	=	9, 299178	9, 272196
$l\,T^2$	=	3, 111882	
$l\,4\,m^2$	=	11, 471165	
$l\,4\,m^2\,T^2$	=	14, 583047	
$l\,y^2z^2$	=	20, 619216	20, 661934
add. 2 l fin ϕ	=	8, 598356	8, 544392
		19, 217572	19, 206326
fubtr. $l\,4\,m^2\,T^2$	=	14, 583047	14, 583047
l Partis prior.	=	4, 634525	4, 623279
$l\,\sqrt{y\,z}$	=	5, 154804	5, 165484
add. 2 l fin ϕ	=	8, 598356	8, 544392
		3, 753160	3, 709876
fubtr. l 3		0, 477121	0, 477121

l part.

		A	B
l part. poſt.	=	3, 276039	3, 232755
Pars prior	=	43105	42003
Pars poſt.	=	1888	1709
b	=	44993	43712
y	=	114317	117221
z	=	178442	182794
$y - b$	=	69324	73509
$z - b$	=	133449	139082
A *l* $(z-b)$	=	5, 125316	5, 143270
ſubtr. *l* $(y-b)$	=	4, 840884	4, 866341
		0, 284432	0, 276929
add. *l* $\frac{y}{z}$	=	9, 806614	9, 807045
		0, 091046	0, 083974
ſubtr. *l* ſin φ	=	9, 299178	9, 272196
l part. ſubtr.	=	0, 791868	0, 811778
Pars ſubtr.	=	6, 19253	6, 48303
a cot φ	=	4, 92077	5, 24883
— tang *v*	=	1, 27176	1, 23420
Ergo 180 — *v*	=	51,49,18	50,59,3
& anomalia vera *v*	=	128,10,42	129, 0, 57
ideoque ang ASH	=	4ˢ,8,10,42	4, 9, 0, 57
add. HSN	=	24,18, 6	22, 37,11
Diſt. Perihelii a Nodo ☊:		5 , 2, 28, 48	5 , 1, 38, 8
Diſt ☋ a perihelio	=	27, 31, 12	28, 21, 52
l — coſ *v*	=	9, 791067	9, 799021
l y	=	5, 058111	5, 069006
l — *y* coſ *v*	=	4, 849178	4, 868027
add. *l b*	=	4, 653145	4, 640601
l — Numer:		9, 502323	9, 508628

—*y*coſ*v*

		A	B
$-y \cos v$	=	70661	73795
add. $y - b$	=	69324	73509
$-b + y - y \cos v$	=	139985	147304
l − Denom.	=	5, 146081	5, 168214
a l − Num.	=	9, 502323	9, 508628
$l\,a$	=	4, 356242	4, 340414
Dist. Perih. a Sole a	=	22711	21898
hinc $2a$	=	45422	43796
b	=	44993	43712
$2a - b$	=	429	84
$l\,(2a - b)$	=	2, 632458	1, 924280
subtr. $l\,b$	=	4, 653145	4, 640601
$l\,n$	=	7, 979313	7, 283679
$l\,a\,a$	=	8, 712484	8, 680828
subtr. $l \sqrt{b}$	=	2, 326572	2, 320300
		6, 385912	6, 360528
subtr. $l\,m$	=	5, 434553	5, 434553
$l \frac{a\,a}{m \sqrt{b}}$	=	0, 951359	0, 925975
Ob v	=	128, 10, 42	129, 0, 57
erit $\frac{1}{2}\,v$	=	64, 5, 21	64, 30, 28
& $l\,t$	=	0, 313536	0, 321655
$l\,t^2$	=	0, 627072	0, 643310
$l\,t^3$	=	0, 940608	0, 964965
$l\,t^5$	=	1, 567680	1, 608275
$l\,t^7$	=	2, 194752	2, 251585
$l\,t^9$	=	2, 821824	2, 894895
$l\,n\,t^5$	=	9, 546993	8, 891954
$l\,n^2\,t^5$	=	7, 526306	6, 175633

$l\,n^2\,t^7 =$

		A	B
$ln^2 t^7$	=	8, 153378	6, 818943
$ln^3 t^7$	=	6, 132691	4, 102622
$ln^3 t^9$	=	6, 759763	4, 745932
Ergo t	=	2, 05843	2, 09727
$+\frac{1}{3}t^3$	=	2, 90728	3, 07499
		4, 96571	5, 17226
subt. $\frac{2}{5}nt^5 + \frac{4}{7}n^3t^7 + \frac{4}{9}n^3t^9$	=	0, 14126	0, 03119
		4, 82445	5, 14107
add. $\frac{3}{5}n^2t^5 + \frac{3}{7}n^2t^7 +$ &c.	=	811	9
$t + \frac{1}{3}t^3 +$ &c.	=	4, 83256	5, 14116
$l\,(t + \frac{1}{3}t^3 +$ &c.	=	0, 684177	0, 711062
add. $l\,\frac{aa}{m\sqrt{b}}$	=	0, 951359	0, 925975
l temp.	=	1, 635536	1, 637037
Temp.	=	43, 205	43, 355
seu		$43^d, 4^h, 55'$	$43^d, 8^h, 31'$
At tertia Observ. Jan.		18, 7, 57	18, 7, 57
Cometa in Perih. Mart.		1, 12,52	1^d,16,28

Orbita ergo Cometæ sequentibus sex momentis determinabitur:

Pro hypoth. G g	=	101000	106000
1. Distantia Perihelii a Sole seu a	=	22711	21898
& $l\,a$	=	4, 356242	4, 340414
2. Semilatus rectum orbitæ seu b	=	44993	43712
& $l\,b$	=	4, 653145	4, 640601

	A	B
3. Cometa per perihelium transiit A. 1744 Mense Martio Temp. medio Berol.	$1^d,12^h,52'$	$1^d,16^h,28'$
4. Distantia Perihelii a nodo ascendente ☊ =	$152°,28',48''$	$151°,38', 8''$
Hinc a Perihelio ad nodum descendentem ☋ = est anomalia vera =	27, 31, 12	28, 21, 52
5. Longitudo heliocentrica Nodi ascend.	$1^s,15°,24',41''$	$1^s,16°,20',45''$
longitudo heliocentrica nodi descendentis =	$7^s,15°,24',41''$	$7^s,16°,20',45''$
6. Inclinatio Orbitæ Cometæ ad Eclipticam	$42°,59',28''$	$48°,31',29''$

Mox autem apparebit veram Cometæ orbitam intra hos duos limites tam parum a se invicem discrepantes contineri.

Computetur ergo ex utroque limite locus cometæ ad tempus observationis factæ d. 18 Febr. quæ adhuc ante appulsum ad perihelium contigit. Quæratur igitur primo intervallum temporis inter transitum cometæ per perihelium & momentum observationis, idque in diebus, dieique partibus decimalibus exprimatur, quod deinde vocetur = T.

	A	B
Perihel. Cometæ Mart.	$1^d,12^h,52'$	$1^d,16^h,28'$
subtrahatur Febr.	18 , 6, 43	18 , 6, 43
	12 , 6 , 9	12 , 9, 45
Ergo est T =	12, 2562	12, 4062
& lT =	1, 088355	1, 093639

Ponatur anomalia vera huic tempori reſpondens $= v$, ſitque tang. $\frac{1}{2} v = t$, erit $T = \frac{aa}{m\sqrt{b}} \left(t + \frac{1}{3}t^3 - \frac{2}{5}n\,t^5 + \frac{3}{5}n^2 t^5 \text{ \&c.} \right)$ Quoniam orbita parum a parabola diſcrepat, quæratur ex tabula motus in parabola θ, ut ſit $\theta + \frac{1}{3}\theta^3 = \frac{m\sqrt{b}}{aa}.T$ ſeu areæ parabolicæ, quo valore ipſius θ invento erit

$$\theta + \tfrac{1}{3}\theta^3 = t + \tfrac{1}{3}t^3 - \tfrac{2}{5}n\,t^5 + \tfrac{3}{7}n^2 t^7 - \text{\&c.} + \tfrac{3}{5}n^2 t^5 - \tfrac{4}{7}n^3 t^7 +$$

& quia t vehementer parum a θ diſcrepat, ponatur $t = \theta + q$ erit $o = q + \theta\theta q - \frac{2}{5}n\,\theta^5 + \frac{3}{7}n^2\theta^7 - \text{\&c.} + \frac{3}{5}n^2\theta^5 - \frac{4}{7}n^3\theta^7 +$ ideoque fiet proxime

$$q = \frac{(\tfrac{2}{5}n - \tfrac{3}{5}n^2)\theta^5 - (\tfrac{3}{7}n^2 - \tfrac{4}{7}n^3)\theta^7 + (\tfrac{4}{9}n^3 - \tfrac{5}{9}n^4)\theta^9 - \text{\&c.}}{1 + \theta\theta}$$

Ita ergo calculus inſtituatur:

		A	B
$A\ l\ T$	=	1, 088355	1, 093639
ſubtr. $l\ \frac{aa}{m\sqrt{b}}$	=	0, 951359	0, 925975
$l\ (\theta + \frac{1}{3}\theta^3)$	=	0, 136996	0, 167664
Ergo 2 A tang θ	=	91°,3′,20″	93°,41′,55″
& A tang θ	=	45°,31,40	46, 50, 57
& $l\theta$	=	0,008001	0, 028052
$l\theta^2$	=	0,016002	0, 056104
$l\theta^5$	=	0, 040005	0, 140260
$l\,n$	=	7, 979313	7, 283679
$l\,n\,\theta^5$	=	8, 019318	7, 423939
$l\,n^2\theta^5$	=	5, 998631	4, 707618

		A	B
$l n^2 \theta^7$	=	6, 014633	4, 763722
$l n^3 \theta^7$	=	3, 977946	
$+ \frac{2}{5} n \theta^5$	=	0, 004182	0, 001062
$- \frac{3}{5} n^2 \theta^5$	=	59	3
		0, 004123	0, 001059
$- \frac{3}{7} n^2 \theta^7$	=	44	3
Numerator	=	0, 004079	0, 001056
θ^2	=	1, 037535	1, 137900
Denom $1 + \theta^2$	=	2, 037535	2, 137900
l Numerat.	=	7, 610554	7, 023664
l denom.	=	0, 309104	0, 329987
$l q$	=	7, 301450	6, 693677
q	=	0, 002002	0, 000494
θ	=	1, 018595	1, 066725
t	=	1, 020597	1, 067219
Ergo $\frac{1}{2} v$	=	45, 35, 2	46, 51, 45
Anomalia vera v	=	91, 10, 4	93, 43, 30
Perihelium a Nodo aſcend.	=	152, 28, 48	151, 38, 8
Diſt. Cometæ a Nodo	=	61, 18, 44	57, 54, 38

Porro diſtantia cometæ a Sole eſt $= \frac{b}{1 + \frac{b-a}{a} \cos v}$.

Ergo a b	=	44993	43712
ſubtr. a	=	22711	21898
$b - a$	=	22282	21814
$l(b-a)$	=	4, 347954	4, 338735
ſubtr. $l a$	=	4, 356242	4. 340414
erit $l \frac{(b-a)}{a}$	=	9, 991712	9, 998321
add. $l - \cos v$	=	8, 309196	8, 812697

$l a + b$

	A	B
$l\,\frac{a-b}{a}$ cos$\,v$ =	8, 300908	8, 811018
$-\frac{b+a}{a}$ cos$\,v$ =	0, 019994	0, 064717
Denomin.	0, 980005	0, 935282
$l\,b$ =	4, 653145	4, 640601
l Denom. =	9, 991228	9, 970942
l Dist. Com. a Sole =	4, 661917	4, 669659

Resolvendum nunc est triangulum sphæricum ☊ Cc, in quo est C☊ distantia cometæ a nodo ascendente, & angulus ☊ inclinatio orbitæ ad Eclipticam. Erit vero sin Cc = sin ☊ C. sin ☊ & tang ☊ c = tang ☊ C. cos ☊. Fig. 11

	A	B
☊ C =	61, 18, 44	57, 54, 38
ang. ☊ =	42, 59, 28	48, 31, 29
l sin ☊ C =	9, 943122	9, 927995
l sin ☊ =	9, 833710	9, 874621
l tang ☊ C =	9, 776832	9, 802616
l cos ☊ =	10, 261847	10, 202702
l tang ☊ C =	9, 864190	9, 821052
l sin Cc =	10, 126037	10, 023754
Ergo lat. helioc. Cc =	36, 44, 20	39, 24, 10
& ☊ C =	1', 23, 12, 0	1, 16, 34, 0
addatur longitudo ☊ =	1, 15, 24, 41	1, 16, 20, 45
Long. helioc. Cometæ =	3, 8, 36, 41	3, 2, 54, 45
Long. helioc. terræ =	4, 29, 30, 40	1, 29, 30, 40
Ang. TSc =	1, 20, 53, 59	1, 26, 35, 55
summa ang. =	129, 6, 1	123, 24, 5
semi summa =	64, 33, 0	61, 42, 2
l S C =	4, 661917	4, 669659
l sin C S c =	9, 776824	9, 802615
l cos C S c =	9, 903833	9, 888012

Fig. 12.

	A	B
l Cc =	4, 438741	4, 472274
l Sc =	4, 565750	4, 557671
a lST =	4, 995309	4, 995309
l tang: =	10, 429559	10, 437638
ang:	69, 35, 57	69, 56, 42
subtr.	45	45
	24, 35, 57	24, 56, 42
l tang:	9, 660692	9, 667583
l tang $\frac{1}{2}$ summæ:	10, 322480	10, 268869
l tang semid:	9, 983172	9, 936452
semi diff:	43, 53, 25	40, 49, 25
semi summa:	64, 33, 0	61, 42, 2
ang. STc =	20, 39, 35	20, 52, 37
Addatur longitudo Solis	10, 29, 30, 40	10, 29, 30, 40
Longitudo Cometæ Geoc.	11, 20, 10, 15	11, 20, 23, 17

Cum igitur longitudo cometæ observata sit: 11′, 19°, 57′, 0″ vera orbita extra hos limites cadere videtur, ita ut statui deberet Gg = 96000. Videamus ergo quoque latitudinem, est autem $Tc = \frac{Sc \sin TSc}{\sin STc}$.

l Sc =	4, 565750	4, 557671
sub l sin STc =	9, 547550	9, 551890
	5, 018200	5, 005781
add l sin TSc =	9, 889887	9, 921600
l Tc =	4, 908087	4, 927381
a l Cc =	4, 438741	4, 472274
l tang lat:	9, 530654	9, 544893
Lat. Geocentrica:	18°, 44′, 50″	19°, 19′, 30″

Cum

Cum igitur latitudo geocentrica eſſet obſervata 19°, 10′, 56″, orbita vera intra limites hoſce contineri deberet: Videtur autem latitudini plus fidi oportere, quam longitudini; ſin autem utrique æqualiter fidere velimus, aberrationesque in utramque æqualiter diſtribuere, tum hypotheſis A veram orbitam cometæ præbere eſſet cenſenda. Quoniam vero quoque error in obſervationibus tribus aſſumtis ineſſe poteſt, qui etiamſi ſit minimus, tamen notabile diſcrimen in orbitam inferat, plus affirmare non licet, quam hos duos limites proxime ad veritatem accedere.

Interim tamen hoc tuto concludi poſſe videtur, orbitam cometæ neque hyperbolam eſſe, neque parabolam, ſed ellipſin vehementer oblongam: unde ſtatum habebit tempus periodicum. Erit enim diſtantia aphelii a ſole $= \frac{ab}{2a-b}$, tum ſemi axis transverſus $= \frac{aa}{2a-b} = e$

	A	B
Ergo a $2la =$	8, 712484	4, 680828
ſubtr. $l(2a-b) =$	2, 632458	1, 924280
le	6, 080026	6, 756548
$l\sqrt{e} =$	3, 040013	3, 378274
unde $le\sqrt{e} =$	9, 120039	10, 134822
ſubtr. $lc\sqrt{c} =$	7, 500000	7, 500000
Hinc tempus periodicum Cometæ prodiret	1, 620039	2, 634822
annorum:	41, $\frac{69}{100}$	431, $\frac{34}{100}$

Cometa denique in ſuo perihelio propius ad ſolem acceſſit, quam Mercurius, dum in ſuo perihelio verſatur. Hoc enim

tempore

tempore distantia mercurii a sole est = 30740, & quia medium inter utramque hypothesin sumiendo distantia Cometæ in perihelio erat circiter 22000, illa ad hanc rationem proxime habebit ut 7 ad 5.

Definiamus vero adhuc tempus, quo cometa per suum nodum descendentem transierit; quo tempore erat ejus anomalia vera.

	A	B
$v =$	$27°,31',12''$	$28°,21',52''$

Cum autem angulus $\frac{1}{2}v$ fiat satis parvus, valor ipsius $t + \frac{1}{3}t^3 + \&c.$ proxime ex hypothesi parabolæ reperietur fietque.

$l\,(t + \frac{1}{3}t^3 + \&c.) =$	9, 397478	9, 411748
add $l\,\frac{aa}{m\sqrt{b}} =$	0, 951359	0, 925975
	0, 348837	0, 337723
Tempus a Perihelio in diebus =	2, 2327	2, 1764
seu =	$2^d, 5^h, 45'$	$2^d, 4^h, 13'$
Addatur temp. Perih. Mart.	1, 12, 52	1, 16, 28
Cometæ per nodum descendentem transiit A. 1744 Martio	$3^d, 18^h, 37$	$3^d, 20^h, 41'$

Quare cometa die quarto mensis martii circa ortum solis per eclipticam austrum versus est transgressus; motus ergo circa solem fuit celerrimus, quia duobus diebus fere 30 gradus in orbita sua absolvit. Tempus quo Cometa per nodum ascendentem transiit tam accurate definiri nequit, quia ob ingentem anomaliam veram 151°, minimus error in orbita ingens discrimen producere valet. Interim ex limite B colligitur cometa per nodum ascendentem transiisse A. 1743. Mensis Augusti die septimo.

Quan-

Quanquam orbita cometæ hoc modo inventa parum a veritate discrepat, tamen per easdem, quibus usus sum observationes multo accuratius potest determinari eo modo, quem exposui in Miscell. Berol. Volumine VII. ubi ostendi, quemadmodum, si orbita cometæ jam fere sit cognita, ea per observationes corrigi debeat. Fingamus ergo orbitam a veritate parum discrepantem parabolicam; quæ contineatur his conditionibus

	Orbita ficta	sitque orbita vera
Perihel. a sole	22000	$22000 - \alpha$
ratio $b : a$	$2 : 1$	$2 - \frac{\beta}{10000} : 1$
Com. in Perih. Mart.	$1^d, 6^h, 0'$	$1^d, 6^h, \gamma'$
Dist. Perih. a ☊	$151°$,	$151°, \delta'$
Long. hel. ☊	$1^s, 16°$	$1^s, 16° - \varepsilon'$
Incl. Orbitæ	$45°$	$45° + \zeta$.

Ut jam valores litterarum α, β, γ, δ, ε & ζ determinem, sex constituo hypotheses, quarum quælibet unica conditione ab orbita ficta discrepet. sintque:

Hyp. I	Hyp. II	Hyp. III	Hyp. IV	Hyp. V	Hyp. VI
22000	22000	22000	22000	21000	22000
$2 : 1$	$2 : 1$	$2 : 1$	$2 : 1$	$2 : 1$	$2 - \frac{50}{10000} : 1$
$1^d, 6^h$	$1^d, 6^h$,	$1^d, 6^h$	$1^d, 18^h$	$1^d, 6^h$	$1^d, 6^h$
$151°$	$151°$	$152°$	$151°$	$151°$	$151°$
$1^s, 16°$	$1^s, 15°$	$1^s, 16°$	$1^s, 16°$	$1^s, 16°$	$1^s, 16°$
$50°$	$45°$	$45°$	$45°$	$45°$	$45°$

His constitutis eligo quatuor observationes omni cura insti-

tu-

tutas, ad earumque tempora ex ficta orbita atque ex hypothesibus investigo longitudinem & latitudinem geocentricam cometæ; atque ex discrimine singularum hypothesium ab orbita ficta colligi poterit locus cometæ, quem orbita vera esset datura; qui cum observato comparatus dabit æquationem. Quoniam vero tantum sex opus est æquationibus, ex quatuor illis observationibus duas latitudines rejiciamus, quippe quæ per reliqua sponte determinantur. Hoc modo cum absolvissem calculum satis molestum, quem ob prolixitatem hic prætermitto, sex sequentes æquationes sum adeptus:

I. Ex longitudine Dec. 13^d, 8^h, $40'$ observata:

$$483\zeta + 9383\varepsilon - 6366\delta - 46\gamma + 335\alpha + 4220\beta - 41000 = 0$$

II. Ex longitudine Jan. 3^d, 6^h, $17'$ observata

$$1550\zeta + 6116\varepsilon - 3716\delta - 130\gamma + 179\alpha + 3620\beta - 124000 = 0$$

III. Ex latitudine Jan. 3^d, 6^h, $17'$ observata

$$1260\zeta - 1566\varepsilon + 6866\delta - 88\gamma - 495\alpha - 1380\beta - 421000 = 0$$

IV. Ex longitudine Jan. 18^d, 7^h, $57'$ observata

$$1517\zeta + 3883\varepsilon - 1233\delta - 188\gamma + 63\alpha + 2640\beta - 156000 = 0$$

V. Ex latitudine Jan. 18^d, 7^h, $57'$ observata

$$1257\zeta - 1566\varepsilon + 5583\delta - 86\gamma - 459\alpha - 1100\beta - 378000 = 0$$

VI. Ex longitudine Febr. 18^d, 6^h, $43'$ observata

$$1140\zeta - 1817\varepsilon + 1733\delta - 544\gamma - 250\alpha + 560\beta - 131000 = 0$$

Ex his sex æquationibus orientur sex sequentes valores ipsius ζ.

$0 = \zeta + 19{,}426\varepsilon - 13{,}180\delta - 0{,}0952\gamma + 0{,}6936\alpha + 8{,}737\beta - 84{,}886$

$0 = \zeta + 3{,}946\varepsilon - 2{,}397\delta - 0{,}0838\gamma + 0{,}1154\alpha + 2{,}335\beta - 80{,}000$

$0 = \zeta - 1{,}243\varepsilon + 5{,}449\delta - 0{,}0705\gamma - 0{,}3930\alpha - 1{,}100\beta - 334{,}127$

$0 = \zeta + 2{,}559\varepsilon - 9{,}813\delta - 0{,}1239\gamma + 0{,}0415\alpha + 1{,}740\beta - 102{,}834$

$0 = \zeta - 1{,}246\varepsilon + 4{,}442\delta - 0{,}0684\gamma - 0{,}3651\alpha - 0{,}875\beta - 300{,}716$

$0 = \zeta - 1{,}594\varepsilon + 1{,}520\delta - 0{,}4772\gamma - 0{,}2193\alpha + 0{,}291\beta - 114{,}912$

ſubtrahantur ſingulæ æquationes a prima eritque

$0 = 15{,}480\varepsilon - 10{,}783\delta - 0{,}0114\gamma + 0{,}5782\alpha + 6{,}402\beta - 4{,}886$

$0 = 20{,}699\varepsilon - 18{,}629\delta - 0{,}0247\gamma + 1{,}0866\alpha + 9{,}837\beta + 249{,}241$

$0 = 16{,}867\varepsilon - 12{,}367\delta + 0{,}0287\gamma + 0{,}6521\alpha + 6{,}997\beta + 17{,}948$

$0 = 20{,}672\varepsilon - 17{,}622\delta - 0{,}0268\gamma + 1{,}0587\alpha + 9{,}612\beta + 215{,}830$

$0 = 21{,}020\varepsilon - 14{,}700\delta + 0{,}3820\gamma + 0{,}9189\alpha + 8{,}246\beta + 30{,}026$

hinc oriuntur quinque valores pro ε.

$0 = \varepsilon - 0{,}6966\delta - 0{,}0007\gamma + 0{,}0373\alpha + 0{,}4135\beta - 0{,}3156$

$0 = \varepsilon - 0{,}9013\delta - 0{,}0012\gamma + 0{,}0526\alpha + 0{,}4776\beta + 12{,}0587$

$0 = \varepsilon - 0{,}7332\delta + 0{,}0017\gamma + 0{,}0387\alpha + 0{,}4148\beta + 1{,}0641$

$0 = \varepsilon - 0{,}8525\delta - 0{,}0013\gamma + 0{,}0512\alpha + 0{,}4650\beta + 10{,}4410$

$0 = \varepsilon - 0{,}6993\delta + 0{,}0182\gamma + 0{,}0437\alpha + 0{,}3923\beta + 1{,}4284$

Subtrahantur ab ultimo omnes reliqui, atque orientur quatuor ſequentes æquationes.

$0 = 0{,}0189\gamma - 0{,}0027\delta + 0{,}0064\alpha - 0{,}0212\beta + 1{,}7440$

$0 = 0{,}0194\gamma + 0{,}2020\delta - 0{,}0089\alpha - 0{,}0853\beta - 10{,}6303$

$0 = 0{,}0165\gamma + 0{,}0339\delta + 0{,}0050\alpha - 0{,}0225\beta + 0{,}3643$

$0 = 0{,}0195\gamma + 0{,}1532\delta - 0{,}0075\alpha - 0{,}0727\beta - 9{,}0126$

invenientur ergo quatuor valores pro γ.

$$0 = \gamma - 0,1428\,\delta + 0,3386\,\alpha - 1,1217\,\beta + 92,275$$
$$0 = \gamma + 10,4124\,\delta - 0,4587\,\alpha - 4,4890\,\beta - 547,950$$
$$0 = \gamma + 0,2054\,\delta - 0,3030\,\alpha - 1,3636\,\beta + 22,078$$
$$0 = \gamma + 7,8564\,\delta - 0,3846\,\alpha - 3,7282\,\beta - 162,485$$

Subtrahantur prima ac tertia a ſecunda itemque prima a quarta.

$$0 = 10,5552\,\delta - 0,7973\,\alpha - 3,3673\,\beta - 640,225$$
$$0 = 10,2070\,\delta - 0,7617\,\alpha - 3,1254\,\beta - 570,028$$
$$0 = 7,9992\,\delta - 0,7232\,\alpha - 2,6065\,\beta - 554,460$$

Unde tres valores ipſius δ deducuntur:

$$\delta = 0,07554\,\alpha + 0,31902\,\beta + 60,655$$
$$\delta = 0,07462\,\alpha + 0,30620\,\beta + 55,846$$
$$\delta = 0,09041\,\alpha + 0,32585\,\beta + 69,315$$

ſubtrahatur medius ab utroque, eritque:

$$0 = 0,00092\,\alpha + 0,01282\,\beta + 4,809$$
$$0 = 0,01579\,\alpha + 0,01965\,\beta + 13,469$$

hincque bini valores pro β naſcuntur:

$$\beta = -0,07176\,\alpha - 375,118$$
$$\beta = -0,80356\,\alpha - 685,492$$

ſubtrahatur poſterior a priori eritque

$$0,73180\,\alpha + 310,374 = 0$$

Hinc

unde	$\alpha = -424,125$	$l-\alpha = 2,627493$
hincque	$\beta = -344,680$	$l-\beta = 2,537416$
	$\delta = -81,345$	$l-\delta = 1,910331$
	$\gamma = -346,889$	$l-\gamma = 2,540190$
	$\varepsilon = +101,755$	$l+\varepsilon = 2,007555$
	$\zeta = +311,44$	$l+\zeta = 2,493374$

Hinc ergo ob β negativum fiet orbita cometæ hyperbola ſequentibus ſex conditionibus determinata.

Diſtantia perihelii a Sole $a =$	22424
ratio $b : a =$	$2+\frac{344}{10000} : 1$
Ergo ſemilatus rectum $b =$	45619
Cometa per Perihelium tranſiit A. 1744. Mart.	$1^d, 0^h, 14'$
Diſt. Perihelii a ☊	$149°, 39'$
Long. helioc. Nodi ☊	$1^s, 17°, 41'$
Inclinatio orbitæ	$50°, 11'$

Patet autem has determinationes maxime pendere a bonitate obſervationum, in quibus ſi vel minimum eſſet aberratum, orbita, quæ hic prodiit hyperbola, facile in ellipſin transmutari potuiſſet. Tum vero calculus quoque ſumma cura inſtitui debet, ita ut in locis, quæ ſingulæ hypotheſes præbent, ne minuta quidem ſecunda negligantur. Tantum laborem autem, niſi in obſervationibus tantam fiduciam collocare liceat, ſuſcipere operæ non eſt pretium. Quare mihi quidem methodum oſtendiſſe ſufficiat, cujus ope, ſi accura-

curatissimæ observationes suppetant, veræ orbitæ natura investigari queat; quod negotium aliis expediendum relinquo.

Quoniam ergo ob defectum plurium observationum satis exactarum circa orbitæ naturam, utrum sit in se rediens an in infinitum excurrens, nihil concludere licet, finem huic dissertationi imponam, postquam quædam de hujus cometæ cursu tam observato quam futuro monuero. Primum igitur iste cometa neque eclipticam neque æquatorem trajecisse visus est, sed quamdiu apparuit tam latitudinem quam declinationem habuit borealem, interim tamen si ad ejus tempus periodicum spectemus, quod si ullum est, plurium seculorum esse debet. Semestre tantum spatium in hemisphærio boreali est versatus, reliquo vero tot annorum spatio perpetuo in regione cœli australi est commoratus. Deinde a 7 die Augusti, quo per nodum ascendentem transiit usque ad Februarii diem 25 ab ecliptica recessit, hinc vero cursu satis celeri jam die Martii 4to per nodum descendentem est transgressus, quæ ingens anomalia certe cum nulla alia theoria, præter Neutonianam, consistere potest. Sub finem ergo apparitionis ejus via apparens vehementer a circulo cœli maximo deflexit: unde manifestum est, hunc cometam non in plano per centrum terræ transseunte esse motum. Cum in nodo descendente ver-

saretur

ſaretur, ſoli quidem propior fuit quam Mercurius, ab hoc vero tam parum fuit remotus, ut, ſi vim attractivam pro ratione molis habuerit, notabilis perturbatio in motu mercurii oriri debuiſſet: hoc enim tempore longitudo Cometæ heliocentrica erat, in Scorpii gradu 15, Mercurius vero in ejusdem ſigni gradu 26 hærebat; atque corpus cometæ, ſi diametrum apparentem eo tempore, quo in diſtantia ſolis a nobis erat remotus, ſtatuamus 1′, plusquam tricies corpus terræ ſuperavit. Quamobrem operæ pretium erit inveſtigare, utrum mercurius adhuc in motu ſuo cum tabulis aſtronomicis conſentiat, an vero perturbationem a cometa ſit paſſus.

Poſt Februarii diem ultimum, quo cometa adhuc ante ſolis ortum erat conſpicuus, penitus evanuit, cum ob ſolis vicinitatem, tum ob diminutam ejus latitudinem borealem. Quia enim poſt diem Martii quartum in hemiſphærium auſtrale proceſſit, ante ortum ſolis non amplius ſupra horizontem noſtrum aſcendit. Quibus autem in cœli locis poſtea verſetur, ex his inventis ſatis accurate indicari poterit. Sic Aprilis die 15 iterum erit directus, & apparebit in 8vo Arietis gradu cum latitudine auſtrali 30° fere, & cum diſtantia a terra futura ſit paulo major quam ſolis, incolis terræ auſtralibus adhuc erit conſpicuus; qui poſt quartum diem martii hunc cometam ante ſolis ortum eximio

ſplendore

ſplendore videre debuerunt. Quareſi in his regionibus commorarentur Aſtronomi, diu adhuc poſt nos eundem cometam obſervare potuiſſent. Atque ſi eum exquiſitis tubis proſequerentur, fortaſſe ultra menſem Julium conſpicere poſſent, namque primo die Julii ejus longitudo eſſe debebit ♈ 27°, cum latitudine auſtrali 48°, ſexto autem die ſeptembris longitudo iterum erit ♈ 3°, cum latitudine 53°, ejus vero diſtantia a terra ſe habebit ad diſtantiam ſolis ut $2\frac{1}{2}$ ad 1, unde niſi per bona teleſcopia vix ſpectari poterit. Hujusmodi autem obſervationes in regionibus terræ auſtralibus factæ maxime eſſent optandæ, cum ex iis facile omnia, quæ adhuc circa ejus orbitæ cognitionem deſiderantur, deduci ac ſuppleri poſſent. Utinam hoc tempore in capite bonæ ſpei idoneus aſtronomiæ cultor commoraretur, a quo iſtud ſupplementum expectare liceret.

Additamentum.

I.

Cum ſuperiorem diſſertationem abſolviſſem, atque orbitæ cometæ determinationem ex obſervationibus initio commemoratis deductam ad Academiam Regiam Pariſinam miſiſſem; Celeb. Caſſinus, ut deſiderio meo theoriam per obſervationes comprobandi ſatisfaceret, omnes quas fecerat, obſervationes mecum benevole communicavit. Sunt autem ſequentes.

Pariſiis tempore apparente	Longitudo Cometæ obſervata	Latitudo obſervata B.
1743. Dec. 21^d, 6^h, 58$''$	0^s, 22°, 23$'$, 0$''$	16°, 18$'$, 57$''$
30, 5, 54	0, 16, 29, 38	17, 12, 55
1744. Jan. 1, 5, 41	0, 15, 19, 35	17, 23, 23
3, 5, 28	0, 14, 11, 18	17, 32, 39
4, 5, 21	0, 13, 38, 11	17, 37, 27
5, 5, 14	0, 13, 5, 57	17, 42, 16
6, 5, 8	0, 12, 34, 44	17, 46, 20
7, 5, 2	0, 12, 3, 12	17, 51, 23
8, 4, 55	0, 11, 33, 8	17, 55, 50
10, 9, 42	0, 10, 24, 34	18, 5, 14
11, 9, 1	0, 9, 58, 40	18, 9, 35
12, 9, 11	0, 9, 31, 15	18, 13, 26
13, 7, 52	0, 9, 5, 30	18, 17, 4
16, 8, 43	0, 7, 45, 15	18, 30, 26
17, 7, 41	0, 7, 20, 58	18, 34, 22
18, 7, 0	0, 6, 56, 46	18, 38, 2

Febr.	1,	7,	55	0,	1,	9,	54	19,	34,	0
	3,	7,	49	0,	0,	18,	26	19,	42,	53
	7,	7,	55	11,	28,	16,	22	19,	53,	54
	10,	7,	17	11,	26,	32,	1	19,	56,	23
	11,	5,	47	11,	25,	52,	41	19,	57,	35
	12,	5,	51	11,	25,	12,	45	19,	56,	4
	13,	5,	39	11,	24,	28,	25	19,	53,	15
	15,	6,	46	11,	22,	46,	47	19,	44,	15
	16,	6,	19	11,	21,	54,	54	19,	36,	0
	17,	6,	30	11,	20,	55,	51	19,	23,	0
	18,	6,	3	11,	19,	54,	0	19,	10,	30
	23,	5,	34	11,	13,	12,	44	16,	41,	3
	24,	5,	47	11,	11,	36,	30	15,	48,	4
	25,	5,	22	11,	9,	52,	46	14,	39,	7
	29,	18,	44	11,	2,	31,	59	6,	28,	21

2. Primo quidem intuitu noſtra hujus cometæ theoria his obſervationibus mirifice confirmatur. Cum enim cometa ab initio apparitionis usque ad diem 18mum Februarii latitudinem fere eandem tenuiſſet, atque etiam ſecundum longitudinem ſatis tarde eſſet motus, hinc quaſi ſubito curſum eclipticam verſus inflectit, ſimulque ſecundum longitudinem motu citatiori progreditur. Ita igitur facile perſpicitur, quomodo cometa jam ante quartum Diem Martii ad eclipticam pervenire potuerit, uti theoria poſtulabat. Quin etiam ſi ad iſta obſervationum tempora ex orbita ante inventa cometæ loca ſupputentur, vix notabile diſcrimen deprehenditur: hincque adeo certe nobis concludere licet, affectiones orbitæ cometicæ detectas ad veritatem proxime accedere.

3. At

3. At vero quod caput rei est, & unde hoc additamentum est natum, istæ observationes adhiberi possunt, ad exiguas illas aberrationes, quibus orbita supra determinata adhuc laborat, emendandas, atque ad verum cometæ motum accuratissime definiendum. Hunc in finem ea ipsa methodus, quæ in hac dissertatione est exposita, hic quoque in usu vocari posset; quia autem tria loca cometæ admodum vicina requirit, multitudo & insigne observationum intervallum, quæ duæ res tamen maxime orbitis cometarum cognoscendis inserviunt, parum subsidii afferrent. Neque etiam hac methodo nunc amplius indigemus, cum orbita jam prope sit nota, atque aliam dederimus methodum orbitas jam vero proximas per plures observationes corrigendi.

4. Interim tamen & hæc methodus, qua jam sumus usi ad cursum cometæ ex observationibus accuratius determinandum, non contemnendas habet difficultates. Primo enim nimis longum ac tædiosum calculum requirit, ac deinde multitudo incognitarum, quas ex aberrationibus definiri oportet non solum operationem reddit molestissimam, sed etiam ob tot quantitates neglectas, minus certam. Quanquam enim istæ quantitates tam sunt parvæ, ut singulæ sine errore rejici possent, tamen eæ conjunctim sumtæ, errorem notabilem procreare possunt. Hanc ob causam in aliam methodum inquisivi, cujus ope orbita cometæ jam propemodum cognita non solum facilius & expeditius emendari queat, sed quæ etiam pauciores in-

cognitas ad negotium abſolvendum requirat. Cum igitur hoc deſideratum aliquamdiu animo volviſſem, ſequentem methodum ſum aſſecutus, cujus ope ex datis tribus tantum cometæ obſervationibus quibuscunque orbita jam pæne cognita exactiſſime aſſignari poteſt.

5. Ne igitur ſex illa capita, quibus orbita cometæ determinatur, ſimul in computum inducantur, ex iis tantum aliquot tanquam cognita aſſumamus, unde ex dato quovis loco obſervato cometæ locus heliocentricus concludi queat. Hoc autem fieri poteſt ex poſitione lineæ nodorum, & inclinatione orbitæ ad eclipticam: his enim duobus tantum capitibus cognitis, ex quovis loco cometæ geocentrico ejus locus heliocentricus atque diſtantia a ſole definiri poteſt, quæ methodus pro planetis jam pridem eſt nota & paſſim explicata; interim tamen ob nimis parvam inclinationem orbitarum planetarum, niſi obſervationes ſint fere ad minuta ſecunda exactæ, tuto adhiberi nequit. Si enim inclinatio plane evaneſceret, tum hinc nihil prorſus concludi poſſet: ex quo facile colligitur, quo minor ſit orbitæ cujuspiam inclinatio, eo minus tuto hanc methodum adhiberi poſſe. Cum igitur cometarum orbitæ plerumque magnos angulos cum ecliptica conſtituant, ad eos hæc methodus maxime eſt accommodata, atque feliciſſimo ſucceſſu uſurpari poterit.

6. Si ergo aſſumamus poſitionem lineæ nodorum atque inclinationem orbitæ ad eclipticam eſſe datas, ex quovis come-

tæ loco obſervato ejus locum verum in cœlo aſſignare poterimus, ope ſequentis problematis. Quanquam autem hæ duæ res tantum prope ſunt cognitæ, ſolutio tamen iſtius problematis maximam afferet utilitatem, quoniam deinceps monſtrabo, quemadmodum per tres obſervationes eædem accuratius determinari queant.

Problema I.

Datis orbitæ cometæ interſectione cum ecliptica & ejus inclinatione ad eam, ad datum quodvis tempus, quo longitudo & latitudo cometæ geocentrica eſt obſervata, ejus locum verum heliocentricum cum ejus a ſole diſtantia invenire.

Solutio.

7. Computetur ad tempus propoſitum locus ſolis, ſitque diſtantia terræ a ſole $ST = c$. Denotet ſcilicet S ſolem, T terram, ſitque SN linea nodorum, cujus poſitio cum ſit data, dabitur angulus TSN qui ponatur $= s$. Deinde ducatur ſecundum longitudinem cometæ obſervatam recta TN lineam nodorum interſecans in N, eritque angulus STN differentia longitudinum ſolis & cometæ, & propterea datus. Poſito ergo angulo hoc $STN = t$, erit angulus $SNT = 180° - s - t$, qui ponatur $= n$. Hinc in triangulo STN ob omnes angulos cum latere $ST = c$ cognitos, reperietur $TN = \frac{ST \sin s}{\sin n}$ & $SN = \frac{ST \sin t}{\sin n}$. His præparatis ſit C locus cometæ Fig. 1

 verus

verus in sublimi positus, unde ad planum eclipticæ TSN demittatur perpendiculum Cc, eritque CTc latitudo cometæ geocentrica, quæ si ponatur $= p$, erit ang. $CTc = p$. Tum si ex c ad SN ducatur perpendicularis cP jungaturque recta CP, repræsentabit angulus CPc inclinationem orbitæ ad Eclipticam, ponatur ea $= i$, eritque $CPc = i$. Ex his considerationibus determinabitur punctum c. Posita enim $cN = x$; & $TN = \frac{c \sin s}{\sin n} = a$, erit $Tc = a - x$. Deinde in triangulo cNP fiet $cP = x \sin n$, & ex triangulo CPc fiet $Cc = x \sin n. \tang i$. At ex triangulo CTc sequitur $Cc = (a-x) \tang p$; quibus duobus valoribus æquatis reperitur $x = \frac{a \tang p}{\tang p + \sin n. \tang i} = cN$ & $Tc = \frac{a \sin n. \tang i}{\tang p + \sin n. \tang i}$ $= \frac{c \sin s. \tang i}{\tang p + \sin n. \tang i}$, hincque $TC = \frac{c \sin s. \tang i}{\sin p + \sin n. \cos p \tang i}$ quæ est distantia cometæ a terra. Porro vero invento $cN = x$, habebitur $PN = x \cos n$ & $cP = x \sin n$; cognosceturque $SP = SN - NP$; unde oritur $\tang cSN = \frac{cP}{SP}$, cognitoque angulo cSN erit $Sc = \frac{cP}{\sin cSN} = \frac{SP}{\cos cSN}$.

Tum vero erit $\tang CSc = \frac{Cc}{Sc}$; sicque innotescit latitudo heliocentrica CSc, ex qua efficitur distantia cometæ a sole $SC = \frac{Cc}{\sin CSc} = \frac{Sc}{\cos CSc}$. Denique cum sit $\frac{SP}{SC} = \cos$ CSN,

C S N, hic angulus indicabit elongationem cometæ heliocentricam a nodo N, in sua orbita. Cum igitur planum orbitæ cometæ detur, ex angulo N S C & recta S C definitur locus cometæ verus heliocentricus. Q. E. J.

Coroll. 1.

8. Cum sit $a = \frac{c \sin s}{\sin n}$, erit $x = cN = \frac{c \sin s \operatorname{tang} p}{\sin n (\operatorname{tang} p + \sin n \operatorname{tang} i)}$ & $cP = \frac{c \sin s \operatorname{tang} p}{\operatorname{tang} p + \sin n \operatorname{tang} i}$; ideoque $CP = \frac{c \sin s \operatorname{tang} p : \operatorname{cos} i}{\operatorname{tang} p + \sin n . \operatorname{tang} i}$. Tum vero erit $PN = \frac{c \sin s \operatorname{tang} p \cot n}{\operatorname{tang} p + \sin n \operatorname{tang} i}$, qui valor subtractus ab $SN = \frac{c \sin t}{\sin n}$ relinquit $SP = \frac{c (\sin t \operatorname{tang} p + \sin n . \sin t . \operatorname{tang} i - \operatorname{cos} n . \sin s \operatorname{tang} p)}{\sin n (\operatorname{tang} p + \sin n \operatorname{tang} i)}$. At ob $\sin t = \sin n \operatorname{cos} s + \operatorname{cos} n . \sin s$, habebitur $SP = \frac{c (\operatorname{cos} s \operatorname{tang} p + \sin t \operatorname{tang} i)}{\operatorname{tang} p + \sin n \operatorname{tang} i}$.

Coroll. 2.

9. Quia igitur $\frac{CP}{SP}$ dat tangentem anguli CSN, erit $\operatorname{tang} CSN = \frac{\sin s \operatorname{tang} p}{\operatorname{cos} i \operatorname{cos} s \operatorname{tang} p + \sin t \sin i}$. Hinc ergo erit $\cot CSN = \frac{\operatorname{cos} i}{\operatorname{tang} s} + \frac{\sin t \sin i}{\sin s \operatorname{tang} p}$. Ex solis ergo angulis cognitis s, t, i, & p invenitur elongatio cometæ a nodo seu angulus CSN.

Coroll. 3.

Coroll. 3.

10. Invento angulo CSN, ponatur iste angulus CSN $= m$, ut sit $\cot m = \frac{\cos i}{\operatorname{tang} s} + \frac{\sin t \sin i}{\sin s \operatorname{tang} p}$, fiet $\frac{\sin i}{\operatorname{tang} p}$ $= \frac{\sin s \cot m}{\sin t} - \frac{\sin s \cos i}{\sin t \operatorname{tang} s}$. Cum igitur inventum sit CP $= \frac{c \sin s \operatorname{tang} p : \cos i}{\operatorname{tang} p + \sin n. \operatorname{tang} i} = \frac{c \sin s}{\cos i + \sin n. \sin i : \operatorname{tang} p}$, si hic valor ille loco $\frac{\sin i}{\operatorname{tang} p}$ substituatur, erit CP $=$ $\frac{c \sin t}{\cos i \cos n + \sin n. \cot m}$.

Coroll. 4.

11. Quia porro $\frac{CP}{\sin m}$ dat distantiam cometæ a sole SC erit $CS = \frac{c \sin t}{\sin m. \cos n. \cos i + \sin n. \cos m}$. Hincque ex distantia terræ a sole $ST = c$, & angulis m, n, i & t reperitur distantia cometæ a sole CS. At vero ob $\sin m. \cos n = \frac{1}{2} \sin (m + n) + \frac{1}{2} \sin (m - n)$; $\cos m. \sin n = \frac{1}{2} \sin (m + n) - \frac{1}{2} \sin (m - n)$ atque $\frac{1 + \cos i}{2} = (\cos \frac{1}{2} i)^2$ & $\frac{1 - \cos i}{2} = (\sin \frac{1}{2} i)^2$ erit

$$CS = \frac{c \sin t}{\sin (m + n) (\cos \frac{1}{2} i)^2 - \sin (m - n) (\sin \frac{1}{2} i)^2}.$$

Coroll. 5.

12. Commodissime ergo calculus instituetur, quærendo primum has quantitates:

cot

$$\cot CSN = \frac{\cos i}{\tan s} + \frac{\sin t.\ \sin i}{\sin s\ \tan p}$$

$$\frac{ST}{CP} = \frac{\cos i}{\sin s} + \frac{\sin n.\ \sin i}{\sin s.\ \tan p}$$

Quibus inventis erit $SC = \frac{CP.}{\sin CSN}$.

Coroll. 6.

13. Inventa autem recta CP statim innotescet distantia cometæ a terra CT. Cum enim sit $\sin p = \frac{Cc}{cT}$ & $\sin i = \frac{Cc}{CP}$ erit $\frac{\sin p}{\sin i} = \frac{CP}{CT}$ ideoque $CT = \frac{CP.\ \sin i}{\sin p}$. Ad institutum autem nostrum distantia cometæ a terra non indigemus

Coroll. 7.

14. Si cometa in ipsa ecliptica observetur, ita ut ejus latitudo geocentrica p sit nulla, fiet $\cot CSN = \infty$ ideoque ipse angulus CSN evanescet. Versabitur ergo Cometa in ipso puncto N, eritque ejus distantia a sole $= \frac{c \sin t}{\sin n}$.

Coroll. 8.

15. Si terra in ipsa linea nodorum versetur, fiet angulus $TSN = s = o$. Quo casu & CP & angulus CSN evanescere videtur. At vero hoc casu quoque fiet $\tan p + \sin n.\ \tan i = o$, ita ut nihilominus lineæ cP & CP finitum valorem retineant. Quantæ autem sint istæ lineæ una cum angulis hoc casu definiri nequit. Quanquam autem hujusmodi observatio-

nes ad præsens institutum inutiles videntur, tamen eæ inservient, si positio lineæ nodorum jam fuerit cognita, ad inclinationem orbitæ accurate definiendam, erit enim tang $i = -\frac{\text{tang } p}{\text{sin } n}$: unde ad orbitas planetarum determinandas insignis usus redundat.

Coroll. 9.

16. Si fiat angulus $n = o$, quod evenit, si longitudo cometæ geocentrica congruat cum longitudine nodi heliocentrica, erunt sinus angulorum s & t æquales, habebiturque distantia cometæ a terra $TC = \frac{c \text{ sin } s \text{ tang } i}{\text{sin } p}$, & cot $CSN = \frac{\text{cos } i}{\text{tang } s} + \frac{\text{sin } i}{\text{tang } p}$ atque $\frac{ST}{CP} = \frac{\text{cos } i}{\text{sin } s}$ seu $CP = \frac{\text{sin } s}{\text{cos } i} \cdot ST$.

Coroll. 17.

17. Si cometa in oppositione vel conjunctione solis secundum longitudinem fuerit observatus, erit sin $t = o$: hoc ergo casu habebitur cot. $CSN = \frac{\text{cos } i}{\text{tang } s}$, atque $\frac{ST}{CP} = \frac{\text{cos } i}{\text{sin } s} + \frac{\text{sin } i}{\text{tang } p}$ ob sin n = sin s.

Coroll. II.

18. Si tandem inclinatio orbitæ cometæ ad eclipticam evanesceret, ut esset $i = o$: tum necesse est ut quoque latitudo

titudo obſervata p ſit nulla, hocque ergo caſu ob fractionem $\frac{\sin i}{\tan p}$ indefinitam nihil concludi poſſet.

Coroll. 12.

19. Niſi ergo tempore obſervationis locus terræ fuerit prope lineam nodorum, ex loco cometæ geocentrico obſervato ſemper ejus locus verus, hoc eſt elongatio ejus heliocentrica ab altero nodo una cum ejus diſtantia a ſole definiri poteſt.

20. Poſitis ergo linea nodorum & inclinatione orbitæ cognitis, ſi tria loca cometæ obſervata ad hunc calculum revocentur, invenientur tria loca cometæ vera in ſua orbita. Quam cum conſtet eſſe ſectionem conicam, cujus alter focus in ipſo centro ſolis ſit ſitus, ex iſtis tribus punctis inventis ſectio conica poterit determinari, quem in finem ſequens problema reſolvemus.

Problema II.

Datis tribus cometæ locis veris, una cum ejus diſtantiis a ſole, invenire ejus orbitam; hoc eſt poſitionem perihelii, ejus a ſole diſtantiam, atque quantitatem lateris recti.

Solutio.

21. Repræſentet planum tabulæ id ipſum planum in quo cometa movetur, in quo ſit S centrum ſolis & recta ☊ S ☋ linea nodorum ſeu interſectio hujus plani cum ecliptica. Sint F, G & H tria cometæ loca vera in ejus orbita, dabunturque primum diſtantiæ $SF = f$, $SG = g$ & $SH = h$; tum Fig. 2.

 vero

vero per methodum præcedentem cognoſcuntur anguli F S ☋, G S ☋ & H S ☋; ex quibus ſit F S G $= \phi$ & F S H $= \Psi$. His datis contemplemur, quæ quæruntur, ſitque A orbitæ perihelium, A S C orbitæ axis, ad quem in F applicata normalis S B exhibebit ſemi-latus rectum. Ponatur A S $= a$, B S $= b$, ſitque angulus A S F $= v$, quæ eſt anomalia vera reſpondens loco F. Erit ergo A S G $= v + \phi$ & A S H $= v + \psi$. At ex ſectionum conicarum proprietatibus erit $f = \frac{ab}{a + (b-a)\cos v}$ unde fit $a = \frac{bf\cos v}{b - f + f\cos v}$. Simili modo ex ſecundo loco G erit $a = \frac{bg\cos(v+\phi)}{b - g + g\cos(v+\phi)}$ atque ex tertio loco H habetur $a = \frac{bh\cos(v+\psi)}{b - h + h\cos(v+\psi)}$. Ex prima & ſecunda æquatione ſequitur $(b-g)f\cos v = (b-f)g\cos(v+\phi) = (b-f)g(\cos v.\cos\phi - \sin v.\sin\phi)$; ac dividendo per $\cos v$ proveniet $(b-g)f = (b-f)g\cos\phi - (b-f)g\sin\phi\,\mathrm{tang}\,v$, unde obtinetur $\mathrm{tang}\,v = \cot\phi - \frac{f(b-g)}{g(b-f)}\,\mathrm{cosec}.\phi = \frac{1}{\mathrm{tang}\,\phi} - \frac{f(b-g)}{g(b-f)\sin\phi}$. Similiter prima æquatio cum tertia conjuncta dabit: $\mathrm{tang}\,v = \frac{1}{\mathrm{tang}\,\psi} - \frac{f(b-h)}{h(b-f)\sin\psi}$. Hisque binis ipſius tang v valoribus coæquatis erit $\frac{1}{\mathrm{tang}\,\phi} - \frac{f(b-g)}{g(b-f)\sin\phi} = \frac{1}{\mathrm{tang}\,\psi} - \frac{f(b-h)}{h(b-f)\sin\psi}$ ſeu

$b-f$

$$\frac{b-f}{f \tang \phi} - \frac{b+g}{g \sin \phi} = \frac{b-f}{f \tang \psi} - \frac{b+h}{h \sin \psi}$$: unde concluditur

$$b = \frac{\frac{1}{\tang \phi} - \frac{1}{\sin \phi} - \frac{1}{\tang \psi} + \frac{1}{\sin \psi}}{\frac{1}{f \tang \phi} - \frac{1}{g \sin \phi} - \frac{1}{f \tang \psi} + \frac{1}{h \sin \psi}}$$, quæ formula

ad calculum ſatis eſt apta, faciliusque adhibebitur, quam conſtructiones geometricæ, quæ de hoc problemate paſſim inveniuntur. Invento igitur ſemi-latere recto b poſitio lineæ abſidum definietur ex angulo v, qui ex alterutra harum æquationum eruetur:

$$\tang v = \frac{1}{\tang \phi} - \frac{f(b-g)}{g(b-f)\sin \phi};\quad \tang v = \frac{1}{\tang \psi} - \frac{f(b-h)}{h(b-f)\sin \psi}$$

Hincque denique aſſignari poterit diſtantia perihelii a ſole $a = \frac{bf \cos v}{b-f+f\cos v}$, ſicque orbita cometæ cognoſcetur. Q. E. J.

Coroll. 1.

22. Si ex æquatione ſupra inventa:

$$(b-g)f = (b-f)g\cos\phi - (b-f)g\sin\phi \tang v$$

loco $\tang v$ eliminemus b, reperitur, $b = \frac{fg - fg\cos\phi + fg\sin\phi\tang v}{f-g\cos\phi + g\sin\phi\tang v}$. Simili vero modo ex tertia obſervatione erit $b = \frac{fh - fh\cos\psi + fh\sin\psi\tang v}{f - h\cos\psi + h\sin\psi\tang v}$

Quibus inter ſe comparatis erit:

 $g(1-\cos\phi)$

$$\begin{aligned}
&g(1-\cos\phi)(f-h\cos\psi) + g(f-h\cos\psi)\sin\phi \operatorname{tang} v\\
-&h(1-\cos\psi)(f-g\cos\phi) + gh(1-\cos\phi)\sin\psi \operatorname{tang} v = v\\
&\qquad\qquad - h(f-g\cos\phi)\sin\psi \operatorname{tang} v\\
&\qquad\qquad - gh(1-\cos\psi)\sin\phi \operatorname{tang} v
\end{aligned}$$

seu $f(g-h)-g(f-h)\cos\phi + h(f-g)\cos\psi + g(f-h)\sin\phi \operatorname{tang} v$ $- h(f-g)\sin\psi \operatorname{tang} v = o$ unde invenitur

$$\operatorname{tang} v = \frac{f(h-g)-g(h-f)\cos\phi + h(g-f)\cos\psi}{h(g-f)\sin\Psi - g(h-f)\sin\phi},$$

Coroll. 2.

23. Si alter angulus ϕ æquetur duobus rectis ita ut rectæ F S, F G in directum sint positæ ob $\sin\phi = o$ & $\operatorname{tang}\phi = o$, fiet $h = \dfrac{\dfrac{1}{\operatorname{tang}\phi} - \dfrac{1}{\sin\phi}}{\dfrac{1}{f\operatorname{tang}\phi} - \dfrac{1}{g\sin\phi}} = \dfrac{\cos\phi - 1}{\dfrac{\cos\phi}{f} - \dfrac{1}{g}}$. At est $\cos\phi = -1$, unde erit $h = \dfrac{2fg}{f+g}$ quæ est eximia sectionum conicarum proprietas jam quidem nota.

Coroll. 3.

24. Expressio autem generalis, quam pro valore semi-lateris recti invenimus, si ad lineas revocetur, sequens suppeditabit theorema, naturam sectionum conicarum non parum illustrans.

Theorema.

Fig. 3. *Si a tribus quibuscunque sectionis conicæ punctis F. G. H ad ejus alterum focum S ducantur rectæ F S, G S, & H S atque duo eorum G & H cum tertio F jungantur rectis G F & H F, ac denique ex G ducatur recta G I K parallela ipsi F S secans rectas H F, H S*

si opus

si opus est productas in I & K; erit G I ad SK + KG − SG uti S F ad semissem lateris recti.

Demonstratio.

25. Vocentur ut ante distantiæ $FS = f$; $GS = g$ & $HS = h$ itemque anguli $FSG = \phi$ & $FSH = \psi$. Demittatur ex S in GIK perpendiculum SL, quod vocetur $= r$. Cum igitur anguli GSL, KSL, sint complementa angulorum ϕ & ψ erit $GL = \frac{r}{\operatorname{tang}\phi}$; $KL = \frac{r}{\operatorname{tang}\psi}$: $SG = \frac{r}{\sin\phi}$ & $SK = \frac{r}{\sin\psi}$. Hinc $SK + KG - SG = \frac{r}{\sin\psi} + \frac{r}{\operatorname{tang}\phi} - \frac{r}{\operatorname{tang}\psi} - \frac{r}{\sin\phi}$. Ducatur KM parallela ipsi HF; erit $SH : SK = SF : SM$, seu $h : \frac{r}{\sin\psi} = f : SM$; unde fit $SM = \frac{fr}{h\sin\psi}$, ideoque $FM = IK = f - \frac{fr}{h\sin\psi} = \frac{fr}{r} - \frac{fr}{h\sin\psi}$. At ob $SG = \frac{r}{\sin\phi} = g$ erit $r = g\sin\phi$, hincque $IK = \frac{fr}{g\sin\phi} - \frac{fr}{h\sin\psi}$. Quare $GI = GK - KI = \frac{r}{\operatorname{tang}\phi} - \frac{r}{\operatorname{tang}\psi} - \frac{fr}{g\sin\phi} + \frac{fr}{h\sin\psi}$. Cum igitur invenerimus esse

$$h = \frac{\frac{1}{\operatorname{tang}\phi} - \frac{1}{\sin\phi} - \frac{1}{\operatorname{tang}\psi} + \frac{1}{\sin\psi}}{\frac{1}{f\operatorname{tang}\phi} - \frac{1}{g\sin\phi} - \frac{1}{f\operatorname{tang}\psi} + \frac{1}{h\sin\psi}} = \frac{\frac{r}{\operatorname{tang}\phi} - \frac{r}{\sin\phi} - \frac{r}{\operatorname{tang}\psi} + \frac{r}{\sin\psi}}{\frac{r}{\operatorname{tang}\phi} - \frac{r}{g\sin\phi} - \frac{r}{\operatorname{tang}\psi} + \frac{fr}{h\sin\psi}} f,$$

erit semi-latus rectum

$h =$

$$b = \frac{SK + KG - SG}{GI} . SF,$$ ideoque $GI : SF = SK + KG - SG$: ſemi-latus rectum. Q. E. D.

27. Hactenus temporum, quæ inter obſervationes effluxerunt nullam habuimus rationem. Nunc igitur cognita orbita ope præceptorum ſupra traditorum definiri poterunt tempora, quibus cometa a perihelio A, ad unumquodque trium punctorum F, G, & H pervenire debuit, hisque inventis prodibunt temporum intervalla inter obſervationes elapſa, quæ ſi cum temporibus obſervatis convenire deprehendantur, hoc certum erit ſignum, poſitionem lineæ nodorum atque inclinationem orbitæ cometæ ad eclipticam recte eſſe aſſumtas. Sin autem hæc tempora per calculum eruta cum obſervatis minus conguant, concludendum erit vel in aſſumtione lineæ nodorum, vel inclinationis vel utriusque eſſe aberratum. Qui error ſi fuerit ſatis parvus, debebit autem eſſe talis, ſi quidem hæc elementa jam per methodum primam proxime noverimus, ex ipſa aberrationis quantitate corrigi poterit. Et quia duo habemus temporum intervalla, quæ cum calculo comparari poſſunt, duæ reſultabunt comparationes, quæ ad utrumque errorem, qui forte ſint commiſſi, emendandum ſufficient. Ita igitur tres cometæ obſervationes ſufficient ad ejus orbitam vere cognoſcendam.

27. Poſtquam ergo ope methodi ante expoſitæ orbita cometæ jam propemodum fuerit determinata, exinde tam poſitio lineæ nodorum quam inclinatio orbitæ ad eclipticam ſatis exacte ad hoc inſtitutum cognoscentur; neque etiam opus erit illum

calculum

calculum summo studio ad finem perducere, sed sufficiet ope constructionum geometricarum hæc momenta saltem circiter determinavisse; quo pacto totus labor mirifice contrahi poterit. Quando igitur eousque tantum fuerit perventum, ut certo sciamus lineam nodorum una cum inclinatione ad eclipticam non multum a veritate abhorrere: statim hac altera methodo uti licebit, quam in sequente problemate explicabo.

Problema III.

Cognitis jam propemodum positione lineæ nodorum & inclinatione orbitæ cometæ ad eclipticam, veram cometæ orbitam ex tribus observationibus exacte determinare.

Solutio.

28. Sit L longitudo nodi puta ascendentis per superiorem methodum inventa, & I sit inclinatio orbitæ ad eclipticam, ibidem eruta. Constituantur tres hypotheses, in quarum prima longitudo nodi assumatur $= L$ & inclinatio $= I$. In secunda hypothesi longitudo nodi denuo statuatur $= L$, inclinatio autem assumatur $= I + \eta$, sumendo pro η unum, duosve gradus, prouti errorem minorem majoremve æstimamus. In tertia hypothesi statuatur longitudo nodi $= L + \lambda$, inclinatio vero maneat $= I$, ubi iterum ipsi λ valor arbitrarius tribui potest, major minorve, prout discrepantiam ipsius L a veritate majorem minoremve judicamus. Quo autem propius jam noverimus valores ipsarum L & I, eo minores poterimus litteras η & λ assumere, dummodo veritas inter tres hypotheses tanquam limites contineatur. His hypothesibus constitutis eligantur tres cometæ observationes quæcunque, quæ quo longius a se invicem fuerint remotæ, eo accuratiorem reddent determinationem: interim tamen evitari debebunt ejusmodi observationes, quæ sint institutæ tum, cum terra prope lineam nodorum versaretur.

Deinde ad tempora harum obſervationum ſupputentur loca ſolis, cum ejus a terra diſtantiis, tum ſecundum problema primum hujus additamenti pro unaquaque hypotheſi definiantur cometæ a ſole diſtantiæ, atque elongationes a linea nodorum. Quo facto ex ſuperiori problemate determinetur orbita cometæ unicuique hypotheſi conveniens, ſicque obtinebuntur tres diverſæ orbitæ, inter quas vera continebitur; Ad quam inveniendam calculo eruantur temporum intervalla, quæ ſecundum quamlibet hypotheſin inter obſervationes binas præterlabi debuiſſent. Sit T tempus inter primam & ſecundam obſervationem, quod ex prima hypotheſi invenitur; $T + p$ tempus quod ex ſecunda, & $T + q$ tempus quod ex tertia hypotheſi eruitur. Tempus autem obſervatum ſit $= T + k$. Si jam ſumamus in orbita cometæ vera eſſe longitudinem nodi accendentis $= L + x$ & inclinationem ad eclipticam $= I + y$, ut res hoc modo ſe habeat.

	Hypoth. I	Hyp. II	Hyp. III	Orbita vera
Longitudo nodi –	L	L	$L + \lambda$	$L + x$
Inclinatio – –	I	$I + \eta$	I	$I + y$
Tempus inter I & II obſervationem	T	$T + p$	$T + q$	$T + k$

Nunc ita ratiocinemur, quoniam ſi inclinationi I augmentum $= \eta$ tribuitur, tempus T incrementum capit $= p$ incrementum y inclinationis dabit temporis incrementum $= \frac{py}{\eta}$. Deinde quoniam longitudinis nodi incrementum λ producit temporis incrementum $= q$, longitudinis L incrementum $= x$ producet temporis incrementum $= \frac{qx}{\lambda}$. Quare ex orbita vera erit intervallum temporis inter primam & ſecundam obſervationem $= T + \frac{py}{\eta} + \frac{qx}{\lambda}$, quod cum

æquale

æquale esse debeat tempori observato T + k: habebitur æquatio $\frac{py}{\eta} + \frac{qx}{\lambda} = k$. Similis æquatio elicietur ex intervallo temporis inter primam & tertiam observationem, atque ex his duabus æquationibus definientur quantitates x & y, hincque innotescent & vera longitudo nodi ascendentis $= L + x$, & vera inclinatio orbitæ ad eclipticam $= I + y$. Denique cum ex comparatione trium assumtarun hypothesium constet, quantum incrementa η & λ cum latus rectum orbitæ b tum distantiam perihelii a sole a, item anomaliam veram primæ observationis v una cum tempore a perihelio ad primam observationem elapso, immutaverint, per regulam auream definientur harum rerum mutationes ex incrementis x & y simul sumtis oriundæ. Hocque modo determinabuntur orbitæ veræ latus rectum, distantia perihelii a sole, elongatio perihelii a loco primæ observationis, ideoque etiam ab altero, ac tandem tempus quo cometa per perihelium transierit. Q. E. J.

Coroll. 1.

29. Cognitis anomaliis veris cometæ in singulis observationibus, tempora, quibus eæ a transitu cometæ per perihelium distant modo supra exposito assignabuntur. Sit enim a distantia perihelii a sole, b semi-latus rectum & v anomalia vera cometæ in observatione proposita. Ponatur $\frac{2a-b}{b} = n$ erit n numerus valde parvus si orbita proxime fuerit parabolica, sitque t = tang $\frac{1}{2}$ v. & quæratur valor sequentis seriei

$$S = t + \tfrac{1}{3}t^3 - \tfrac{2}{5}nt^5 + \tfrac{3}{7}n^2t^7 - \tfrac{4}{9}n^3t^9 + \tfrac{5}{11}n^4t^{11} \quad \&c.$$
$$+ \tfrac{3}{5}n^2t^5 - \tfrac{4}{7}n^3t^7 + \tfrac{5}{9}n^4t^9 - \tfrac{6}{11}n^5t^{11}$$

quo invento erit tempus quæsitum in diebus expressum $= \frac{aa}{m\sqrt{b}}S$, existente $m = 271989,739$ &c. $lm = 5,4345525139$.

Coroll. 2.

30. Si correctiones x & y, quæ hoc modo inveniuntur prodeant nimis magnæ, minus quoque erunt accuratæ. Quod enim posuimus

mus variationes ab incrementis, quas longitudini nodi & inclinationi tribuimus, oriundas his ipsis incrementis esse proportionales, hoc tantum locum habere potest, si incrementa fuerint minima. Interim tamen iste labor non erit inutilis, sed ejus ope cognoscemus positionem lineæ nodorum & inclinationem orbitæ ad eclipticam multo propius, quam assumseramus; sicque exinde novas hypotheses veritati magis consentaneas & inter se propiores formare poterimus. Quo facto si idem calculus, qui in solutione hujus problematis est præscriptus, denuo instituatur, tum exacte orbita cometæ vera cognoscetur, quantum quidem per minimos errores, quibus etiam exactissimæ observationes sunt obnoxiæ, sperare licet.

31. Hac igitur methodo usus sum ad orbitam cometæ priori methodo repertam corrigendam. Quoniam vero observationes, quæ tum erant in potestate, nimis longe a perihelio erant remotæ, inclinatio imprimis ad eclipticam nimis prodiit incerta, quoniam parva mutatio in hypothesi distantiæ cometæ a terra facta differentiam aliquot graduum produxerat. Hanc obrem coactus fui calculum hic traditum bis applicare: atque ex prima operatione statim cognovi inclinationem orbitæ ad eclipticam notabiliter minorem esse, quam supra essem suspicatus, atque 47° vix excedere. Feci igitur pro secunda operatione hypotheses cum inter se tum veritati propiores. Elegi ad hoc institutum ex observationibus a Celeb. Cassino mecum communicatis primam & ultimam, utpote maxime a se invicem remotas, ac propterea orbitam cometæ accuratissime determinantes; cum ultima fere in ipso perihelio sit facta, ubi motus erat velocissimus, ac minima aberratio satis sensibile discrimen producere debebat. Præterea vero elegi observationem penultimam, quæ etsi ratione temporis ultimæ nimis vicina videatur, tamen angulus, quem cometa interea circa solem descripsit, satis est notabilis, ita ut ista tria puncta aptissima videantur ad orbitam determinandam. Hunc igitur calculum integrum, quo secunda operatio constabat, hic apponam, prætermisso priori, quippe cujus ratio ex hoc facile perspicitur.

Tempo-

Tempore medio Pariſino St. novi.

A. 1743.	Long. Cometæ	Lat. Com. boreal.
Dec. 21^d, 6^h, 57′	0^s,22°,23′,0″	16°,18′,57″ = p
Locus Solis.	8, 29, 36, 0	4, 992675 = lS T
t = S T N =	3, 22, 47, 0 =	112°,47′,0″

	Hypoth. I.	Hypoth. II.	Hypoth. III
Incl. ad Ecl. = i =	46°,50′,0″	47°,10′,0″	46°,50′,0″
Long. ☊ helioc. =	1^s,15, 50, 0	1, 15,50,0	1, 16,10,0
Long. Terræ =	2, 29, 36, 0	2, 29,36,0	2, 29,36,0
s = TSN =	43, 46, 0	43, 46, 0	43, 26, 0
t = STN =	112, 47, 0	112, 47, 0	112, 47, 0
$s + t$ =	156, 33, 0	156, 33, 0	156, 13, 0
n = SNT =	23, 27, 0	23, 27, 0	23, 47, 0
A l coſ i =	9, 835134	9, 832425	9, 835134
ſubtr. { l tang s =	9, 981297	9, 981297	9, 976238
{ l ſin s =	9, 839932	9, 839932	9, 837279
l coſ i: tang s =	9, 853837	9, 851128	9, 858896
l coſ i: ſin s =	9, 995202	9, 992493	9, 997855
Ad l tang p =	9, 466453	9, 466453	9, 466453
add. l ſin s =	9, 839932	9, 839932	9, 837279
	9, 306385	9, 306385	9, 303732
ſubtr. a l ſin i =	9, 862946	9, 865302	9, 862946
	0, 556561	0, 558917	0, 559214
add { l ſin t =	9, 964719	9, 964719	9, 964719
{ l ſin n =	9, 599827	9, 599827	9, 605606
l ſin t. ſin i: ſin s. tang p =	0, 521280	0, 523636	0, 523933
l ſin n. ſin i: ſin s. tang p =	0, 156388	0, 158744	0, 164820
coſ i: tang s =	0, 714228	0, 709787	0, 722597
ſin t. ſin i: ſin s. tang p =	3, 321080	3, 339160	3, 341440
cot. CSN =	4, 035308	4, 048947	4, 064037
l cot. CSN =	10, 605876	10, 607342	10, 608958

Ergo CSN =	13°,55′,5″	13°,52′,25″	13°,49′,25′
cos *i* : sin *s* =	0, 989015	0, 982865	0, 995075
sin *n*. sin *i* : sin *s*. tang *p* =	1, 433470	1, 441270	1, 461576
	2, 422485	2, 424135	2, 456651
log : =	0, 384260	0, 384556	0, 390343
a *l* ST =	4, 992675	4, 992675	4, 992675
l CP =	4, 608415	4, 608119	4, 602332
subtr. *l* sin CSN =	9, 381170	9, 379810	9, 378270
l SC =	5, 227245	5, 228309	5, 224062
SC =	168754	169165	167518

Tempore medio Parisino St. novi

A. 1744.	Long. Com.	Lat. Com. Bor.	
Febr. $25^d, 5^h, 36'$	11ˢ 9°,52′,46″	14°,39′,7″ = *p*	
Locus solis	11, 6, 31, 37	4,996003 = *l* ST	
t = STN =	3, 21, 10		
	Hypoth. I	Hypoth. II	Hypoth. III
Incl. ad Ecl. = *i* =	46, 50, 0	47, 10, 0	46, 50, 0
Long. ☊ helioc.	1, 15,50,0	1, 15,50,0	1, 16,10,0
Long. Terræ —	5 , 6,31,40	5 , 6,31,40	5 , 6,31,40
s = TSN =	3, 20,41,40	3, 20,41,40	3, 20,21,40
t = STN =	3,21,10	3,21,10	3,21,10
s + *t* =	3, 24, 2,50	3, 24, 2,50	3, 23,42,50
n = SNT =	65,57,10	65,57,10	66,17,10
A *l* cos *i* =	9, 835134	9, 832425	9, 835134
subtr. { *l* − tang *s* =	10, 422787	10, 422787	10, 430481
subtr. { *l* sin *s* =	9, 971034	9, 971034	9, 971980
l − cos *i* : tang *s* =	9, 412347	9, 409638	9, 404653
l cos *i* : sin *s* =	9, 864100	9, 861391	9, 863154
Ad *l* tang *p* =	9, 417386	9, 417386	9, 417386
add *l* sin *s* =	9, 971034	9, 971034	9, 971980

a *l* sin *i* =

	Hypoth. I	Hypoth. II	Hypoth. III
	9, 388420	9, 388420	9, 389366
a l sin i =	9, 862946	9, 865302	9, 862946
	0, 474526	0, 476882	0, 473580
add. { l sin t =	8, 767038	8, 767038	8, 767038
l sin n =	9, 960571	9, 960571	9, 961689
l sin t. sin i : sin s. tang p =	9, 241564	9, 243920	9, 240618
l sin n. sin i : sin s. tang p =	0, 435097	0, 437453	0, 435269
— cos i : tang s =	0, 258433	0, 256826	0, 253895
sin t. sin i : sin s. tang p =	0, 174407	0, 175356	0, 174027
— cot. CSN =	0, 084026	0, 081470	0, 079868
l — cot. CSN =	8, 924414	8, 910998	8, 902373
Ergo CS☋ =	85, 11,45	85, 20,35	85, 26,0
cos i : sin s =	0, 731308	0, 726760	0, 729716
sin n. sin i : sin s. tang p =	2, 723313	2, 738135	2, 724390
	3, 454621	3, 464895	3, 454106
log =	0, 538400	0, 539690	0, 538335
a lST =	4, 996003	4, 996003	4, 996003
lCP =	4, 458603	4, 456313	4, 457668
subtr. l sin CS☋ =	9, 998471	9, 998564	9, 998619
lSC =	4, 460132	4, 457749	4, 459049
SC =	28849	28691	28777

Tempore medio Parisino St. novi

A. 1744	Long. Cometæ	Lat. Com. bor.	
Febr. 29^d, 18^h, 57$'$	11^s,2°,32$'$,0$''$	6°,28$'$,21$''$	= p
Locus solis =	11, 11, 5,40	4, 996506	= lST
— t = STN =	8°,33$'$,40$''$		

	Hypoth. I	Hypoth. II	Hypoth. III
Incl. ad Ecl. = i =	46°,50$'$,0$''$	47°,10$'$,0$''$	46°,50$'$,0$''$
Long. ☊ helioc. =	1^s,15,50,0	1^s,15,50,0	1^s,16,10,0
Long Terræ =	5, 11, 5,40	5, 11, 5,40	5, 11, 5,40
s = TSN =	3, 25,15,40	3, 25,15,40	3, 24,55,40
— t = STN =	8,33,40	8,33,48	8,33,40

$s + t =$

$s + t$ =	3, 16,42,0	3, 16,42,0	3, 16,22,0
n = SNT =	73,18,0	73,18,0	73,38,0
A l coſ i =	9, 835134	9, 832425	9, 835134
ſubtr. { l tang s =	10, 326180	10, 326180	10, 332758
l ſin s =	9, 956347	9, 956347	9, 957531
l − coſ i: tang s =	9, 508954	9, 506245	9, 502376
l coſ i: ſin s =	9, 878787	9, 876078	9, 877603
Ad l tang p =	9, 054784	9, 054784	9, 054784
add. l ſin s =	9, 956347	9, 956347	9, 957531
	9, 011131	9, 011131	9, 012315
a l ſin s =	9, 862946	9, 865302	9, 862946
	0, 851815	0, 854171	0, 850631
add. { l−ſin t =	9, 172790	9, 172790	9, 172790
l ſin n =	9, 981285	9, 981285	9, 982035
l−ſin t.ſin i: ſin s: tang p =	0, 024605	0, 026961	0, 023421
l ſin n. ſin i: ſin s: tang p =	0, 833100	0, 835456	0, 832666
− coſ i: tang s =	0, 322815	0, 320808	0, 317963
−ſin t. ſin i: ſin s. tang p =	1, 058290	1, 064050	1, 055410
− cot CSN =	1, 381105	1, 384858	1, 373373
l − cot. CSN =	10, 140226	10, 141405	10, 137788
Ergo CS☊ =	35,54,25	35,50,0	36, 3,35
coſ i: ſin s =	0, 756462	0, 751759	0, 754404
ſin n. ſin i: ſin s. tang p =	6, 809265	6, 846300	6, 802460
	7, 565727	7, 598059	7, 556864
log: =	0, 878850	0, 880702	0, 878341
a l S T =	4, 996506	4, 996506	4, 996506
l C P =	4, 117656	4, 115804	4, 118165
ſubtr. l ſin CS☋ =	9, 768246	9, 767474	9, 769840
l S C =	4, 349410	4, 348330	4, 348325
S C =	22357	22301½	22301

A. 1744

A. 1744. Febr.	Hypoth. I	Hypoth. II	Hypoth. III
$29^d, 18^h, 57' \; f =$	22357	$22301\frac{1}{2}$	22301
$l\,SF = lf =$	4, 349410	4, 348330	4, 348325
☋ $SF =$	35,54,25	35,50,0	36, 3, 25
A. 1744. Febr.			
$25^d, 5^h, 36' \; g =$	28849	28691	28777
$l\,SG = lg =$	4, 460132	4, 457749	4, 459049
☋ $SG =$	85, 11,45	85 20, 35	85,26, 0
A. 1743. Dec.			
$21^d, 6^d, 57' \; h =$	168754	169165	167518
$l\,SH = lh =$	5, 227245	5, 228309	5, 224062
☋ $SH =$	166, 4, 55	166, 7, 35	166,10,35
$FSG = \varphi =$	49, 17, 20	49, 30, 35	49, 22, 25
$FSH = \psi =$	130, 10, 30	130, 17, 35	130,7 , 0
$180 - \psi = \chi =$	49, 49, 30	49, 42, 25	49, 53, 0
$l\,\frac{1}{\text{tang}\,\chi} = l\cot\chi =$	9, 926506	9, 928321	9, 925609
$lf =$	4, 349410	4, 348330	4, 348325
$l\,\frac{1}{f\,\text{tang}\,\chi} =$	5, 577096	5, 579991	5, 577284
$l\sin\chi =$	9, 883137	9, 882380	9, 883510
$l\,\frac{1}{\sin\chi} =$	0, 116862	0, 117619	0, 116489
$lh =$	5, 227245	5, 228309	5, 224062
$l\,\frac{1}{h\sin\chi} =$	4, 889617	4, 889310	4, 892427
$l\,\frac{1}{\text{tang}\,\varphi} =$	9, 934737	9, 931342	9, 933437
$lf =$	4, 349410	4, 348330	4, 348325
$l\,\frac{1}{f\,\text{tang}\,\varphi} =$	5, 585327	5, 583012	5, 585112

	Hypoth. I	Hypoth. II	Hypoth. III
$l \sin \phi =$	9, 879674	9, 881109	9, 880225
$l \frac{1}{\sin \phi} =$	0, 120325	0, 118890	0, 119774
$l g =$	4, 460132	4, 457749	4, 459049
$l \frac{1}{g \sin \phi} =$	5, 660193	5, 661141	5, 660725
$1 : \text{tang}\, \chi =$	0, 844319	0, 847854	0, 842576
$1 : \sin \chi =$	1, 308770	1, 311055	1, 307643
$1 : \text{tang}\, \phi =$	0, 860472	0, 853772	0, 857900
	3, 013561	3, 012681	3, 008119
$1 : \sin \phi =$	1, 319246	1, 314892	1, 317573
Numer:	1, 694315	1, 697789	1, 690546
l Num:	0, 228994	0, 229883	0, 228026
$1 : f\, \text{tang}\, \chi =$	377656	380182	377820
$1 : f\, \text{tang}\, \phi =$	384882	382835	384691
$1 : b \sin \chi =$	77556	77502	78060
	840094	840519	840571
$1 : g \sin \phi =$	457292	458291	457852
Denom: =	382802	382228	382719
l Denom: =	5, 582974	5, 582322	5, 582879
l Num: =	0, 228994	0, 229883	0, 228026
$l b =$	4, 646020	4, 647561	4, 645147
$b =$	44261	44418	44172
$g =$	28849	28691	28777
$f =$	22357	22301	23301
$b - g =$	15412	15727	15395
$b - f =$	21904	22117	21871
$l f =$	4, 349410	4, 348330	4, 348325
$l\, 1 : g \sin \phi =$	5, 660193	5, 661141	5, 660725
$l\, b - g =$	4, 187159	4, 196646	4, 187380

$l\,(b - f) =$

	Hypoth. I	Hypoth II	Hypoth III
	4, 196762	4, 206117	4, 196430
$l\,(b-f) =$	4, 340523	4, 344726	4, 339869
	9, 856239	9, 861391	9, 856561
Numer. =	0, 718190	0, 726760	0, 718723
a cot ϕ =	0, 860472	0, 853772	0, 857900
tang v =	0, 142282	0, 127012	0, 139177
l tang v =	9, 153150	9, 103844	9, 143566
v =	8°, 5′,50″	7°,14′,10″	7°,55′,20″
l coſ v =	9, 995648	9, 996527	9, 995835
$l\,f$ =	4, 349410	4, 348330	4, 348325
$l\,f$ coſ v =	4, 345058	4, 344857	4, 344160
$l\,b$ =	4, 646020	4, 647561	4, 645147
l Num: =	8, 991078	8, 992418	8, 989307
f coſ v =	22134	22123	22088
$b-f$ =	21904	22117	21871
Denom: =	44038	44240	43959
l Denom: =	4, 643827	4, 645815	4, 643048
A l Num: =	8, 991078	8, 992418	8, 989307
$l\,a$ =	4, 347251	4, 346603	4, 346259
a =	22246	22213	22195
$2\,a$ =	44492	44426	44390
b =	44261	44418	44172
$2\,a-b$ =	231	8	218
$l\,(2a-b)$ =	2, 363612	0, 903090	2, 338456
$l\,b$ =	4, 646020	4, 647561	4, 645147
$l\,n$ =	7, 717592	6, 255529	7, 693309
$l\,aa$ =	8, 694502	8, 693206	8, 692518
$l\sqrt{b}$ =	2, 323010	2, 323780	2, 322573
	6, 371492	6, 369426	6, 369945
$l\,m$ =	5, 434553.	5, 434553	5, 434553

 l N =

l N =	0, 936939	0, 934873	0, 935392
$\frac{1}{2}v$ =	4°,2′,55″	3, 37, 5	3, 57, 40
$l\,t$ =	8, 849906	8, 800928	8, 840387
$l\,t^3$ =	6, 549718	6, 402784	6, 521161
$l\,3$ =	0, 477121	0, 477121	0, 477121
$l\frac{1}{3}t^3$ =	6, 072597	5, 925663	6, 044040
lN =	0, 936939	0, 934873	0, 935392
l N t =	9, 786845	9, 735801	9, 775779
$l\frac{1}{3}$N t^3 =	7, 009536	6, 860536	6, 979432
N t =	0, 61213	0, 54425	0, 59673
$\frac{1}{3}$N t^3 =	0, 00102	0, 00072	0, 00095
A Perih. ad I.	0, 61315	0, 54497	0, 59768
obſervationem	14^h,42′	13^h,5	14^h,20′
Com. per Perih.	29, 18, 57	29, 18,57	29, 18,57
trans: 1744. Mart.	1^d,9^h,39′	1^d,8^h,2′	1^d,9^h,17′
$\frac{1}{2}v$ =	4, 2, 55	3, 37, 5	3, 57, 40
$\frac{1}{2}\Phi$ =	24, 38, 40	24, 45, 17$\frac{1}{2}$	24, 41, 12$\frac{1}{3}$
$\frac{1}{2}v^1$ =	28, 41, 35	28, 22, 20	28, 38, 50
$l\,t$ =	9, 738250	9, 732451	9, 737421
$l\,t^3$ =	9, 214750	9, 197353	9, 212263
$l\,3$ =	0, 477121	0, 477121	0, 477121
$l\frac{1}{3}t^3$ =	8, 737629	8, 720232	8, 735142
$l\,t^5$ =	8, 691250	8, 662255	8, 687105
$l\,n$ =	7, 717592	6, 255529	7, 693309
$l\,n\,t^5$ =	6, 408842	5, 977784	6, 380414
$l\,n^2t^5$ =	4, 126434	2, 173313	4, 073723
t =	0, 54733	0, 54007	0, 54629
$\frac{1}{3}t^3$ =	0, 05465	0, 05251	0, 05434
	0, 60198	0, 59258	0, 60063
ſubtr. $\frac{2}{5}nt^2$ =	0, 00010	0, 00003	0, 00010

S =

	Hypoth. I.	Hypoth. II.	Hypoth. III
S =	0, 60188	0, 59255	0, 60053
lS =	9, 779510	9, 772725	9, 778535
l N =	0, 936939	0, 934873	0, 935392
A Perih: ad	0, 716449	0, 707598	0, 713927
II Obſerv: dier:	5, 2053	5, 1003	5, 1752
A Per. ad I - -	0, 6131	0, 5449	0, 5977
A I ad II - - - -	4, 5922	4, 5554	4, 5775
$\frac{1}{2}\, v$ =	4, 2, 55	3, 37, 5''	3, 57, 40
$\frac{1}{2}\, \psi$ =	65, 5, 15	65, 8, 37	65, 3, 30
$\frac{1}{2}\, v''$ =	69, 8, 10	69, 45, 40	69, 1, 10
$l\, t$ =	0, 418914	0, 410435	0, 416263
$l\, t^3$ =	1, 256742	1, 231305	1, 248789
$l 3$ =	0, 477121	0, 477121	0, 477121
$l\, \frac{1}{3}\, t^3$ =	0, 779621	0, 754184	0, 771668
$l\, t^5$ =	2, 094570	2, 052175	2, 081315
$l\, n$ =	7, 717592	6, 255529	7, 693309
$l n\, t^5$ =	9, 812162	8, 307704	9, 774624
$l n^2 t^5$ =	7, 529754	4, 563233	7, 467933
$l t^2$ =	0, 837828	0, 820870	0, 832526
$l n^2 t^7$ =	8, 367582	5, 384103	8, 300459
$l\, n$ =	7, 717592	6, 255529	7, 693309
$l n^3 t^7$ =	6, 085174	1, 639632	5, 993768
$l t^2$ =	0, 837828	0, 820870	0, 832526
$l n^3 t^9$ =	6, 923002	2, 460502	6, 826294
$l n$ =	7, 717592	— — — —	7, 693309
$l n^4 t^9$ =	4, 640594		4, 519603
$l n^4 t^{11}$ =	5, 478422		5, 352129
t =	2, 62370	2, 57297	2, 60774
$+ \frac{1}{3}\, t^3$	6, 02034	5, 67785	5, 91110

$-\frac{2}{5} n^2 t^5 =$

	8, 64404	8, 25082	8, 51884
$-\frac{2}{5}\,n\,t^5 =$	0, 25956	0, 00812	0, 23806
	8, 38448	8, 24270	8, 28078
$+\frac{2}{5}\,n^2\,t^5 =$	203		176
	8, 38651	8, 24270	8, 28254
$+\frac{3}{7}\,n^2\,t^7 =$	1000	1	856
	8, 39651	0, 24271	8, 29110
$-\frac{4}{7}\,n^3\,t^7 =$	0, 00007		0, 00005
	8, 39644	8, 24271	8, 29105
$-\frac{4}{9}\,n^3\,t^9 =$	0, 00037		0, 00029
	8, 39607	8, 24271	8, 29076
$+\frac{5}{11}\,n^4\,t^{11} =$	1		1
S $=$	8, 39608	8, 24271	8, 29077
l S $=$	0, 924076	0, 916070	0, 918595
I N $\doteq$	0, 936939	0, 934873	0, 935392
A Perihelio	1 , 861015	1 , 850943	1 , 853987
a III Obſerv. dies	72, 6131	70, 9485	71, 4475
A Perih. ad I -	0, 6131	5, 5449	0, 5977
A I ad III - - -	72, 0000	70, 4036	70, 8498

32. Si igitur correctionem in §. 28. traditam adhibere velimus, fiet: L $=$ 1, 15, 50, 0 & I $= 46°, 50', 0''$ item $\lambda = 20'$ & $\eta = 20'$. Ponatur ergo pro orbita vera Longitudo nodi aſc. ☊ $= 1^s, 15°, 50' + x'$

Inclinatio orbitæ ad Eclipt. $= 46°, 50' + y'$

Conſideremus nunc intervallum temporis inter primam & ſecundam obſervationem, quod revera erat. $4^d, 13^h, 21' = 4, 5562$ dies. Calculo vero ita prodiit

Tempus a I ad II	Hypoth. I	Hypoth. II	Hypoth. III
	4, 5922	4, 5554	4, 5775
T $+ k =$	4, 5562	4, 5922	4, 5922

$k = -360;\ p = -368;\ q = -147$

Cum

Cum ergo ſit $\frac{py}{\eta} + \frac{qx}{\lambda} = k$ erit $360 = 18, 4\,y + 7,3\,x$ ſimili modo cum tempus inter primam & tertiam obſervationem fuerit $= 70^d, 12^h = 7, 5000$, hypotheſes vero habeant.

Tempus	Hyp. I	Hyp. II	Hyp. III
ad I ad III	72, 0000	70, 4036	70, 8498
$T + k =$	70, 5000	72, 0000	72, 0000

$k = -1, 5000$ $p = -1, 5964$ $q = -1, 1502$

Erit ergo $15000 = 798, 2y + 575, 1x$. Ex prima æquatione fit $x = 49, 3 - 2, 5\,y$, qui valor in altera ſubſtitutus dat $6395\,y = 133524$, unde fit: $y = 20', 53''$ hincque erit $x = -3', 54''$. Quare habebitur Pro orbita cometæ vera:

Longitudo nodi aſcendentis — — — 1^s, 15°, 46′, 6″

Inclinatio Orbitæ ad Eclipticam 47°, 10′, 53″

33. Hinc reliqua elementa per interpolationem determinabuntur.

	Hypoth. I	Hypoth. II	Hypoth. III
$a =$	22246	22213	22195
		22246	22246

$p = -33$ $q = -51$

Cum igitur ſit quantitas ad a addenda $k = \frac{py}{\eta} + \frac{qx}{\lambda}$ fiet $k = -34 + 10 = -24$, ideoque vere $a = 22222$.

Deinde

Deinde in erat $b =$	Hypoth. I	Hyp. II	Hyp. III
	44261	44418	44172
		44261	44261

$p = 157$; $q = -89$

ergo addi debet quantitas $k = \frac{py}{\eta} + \frac{qx}{\lambda}$; hoc est $k = 164 + 17 = 181$, unde fit vere $b = 44442$

Cometa per Perih. transiit A. 1744 Mart.	Hyp. I	Hyp. II.	Hyp. III
	$1^d, 9^h, 39'$	$1^d, 8^h, 2'$	$1^d, 9^h, 17'$
		1, 9, 39	1, 9, 39

$p = -1, 37$ $q = -22'$

Hinc $k = \frac{py}{\eta} + \frac{qx}{\lambda} = -1^h, 38'$. Quare revera:

Cometa per perihelium transiit:
A. 1744 Martii $1^d, 8^h, 2'$.

Denique	Hyp. I	Hyp. II	Hyp. III
I. Obs. an. vera	8°, 5′,50″	7, 14, 10	7, 55, 20
I. Obs. a ☋	35, 54, 25	35, 50, 0	36, 3, 35
☋ a Perihel.	27, 48, 35	28, 35, 50	28, 8, 15
		27, 48, 35	37, 48, 35

$p = 47, 15$; $q = 19, 40$

Ergo $k = \frac{py}{\eta} + \frac{qx}{\lambda} = 45', 33''$. Quocirca distantia nodi descendentis a perihelio erit 28°, 34′, 8″. Orbita igitur cometæ vera sequentibus sex elementis definietur.

I. Distantia

I. Diſtantia perihelii a ſole	a	$=$	22222
	$l\,a$	$=$	4, 346783
II. Semi-latus rectum - - -	b	$=$	44442
	$l\,b$	$=$	4, 647793

III. Cometa per Perihelium tranſiit A. 1744 Mart. tempore medio Pariſino. $1^d, 8^h, 2'$

IV. Elongatio nodi deſcendentis ☋ a perihelio ſeu angulus AS☋ $=$ 28°, 34′, 8″

V. Longitudo nodi aſcendentis ☊ - 1ˢ, 15°, 46′, 6″

VI. Inclinatio orbitæ ad Eclipticam 47°, 10′, 53″

34. Orbita ergo hujus cometæ tam parum a parabola diſcrepat, ut ſine ſenſibili errore in calculo pro parabola haberi poſſit. Hinc neceſſe eſt, ut ejus tempus periodicum ſit longiſſimum, plurimisque demum ſeculis abſolvatur; quod quoque experientia confirmatur, cum nullum in tabula Cometarum Hallejana veſtigium inveniamus cujusquam cometæ, cujus orbita ſaltem ratione quorumdam elementorum cum præſenti conveniat. Quoniam autem hæc determinatio cum tribus electis obſervationibus perfecte congruit, dubitare non poterimus, quin eadem & reliquis obſervationibus ſit ſatis factura. Quod ut appareat, ſupputemus ex hac theoria locum cometæ geocentricum pro Februarii 3^d, 8^h, $3\frac{1}{2}'$ tempore medio Pariſino, quo tempore obſervata eſt

longitudo Cometæ	0ˢ, 0°, 18′, 26″
Latitudo Cometæ	19, 42, 53″

Ex tabulis autem solaribus reperitur pro hoc tempore

Longitudo solis		$10^s, 14,26,13$
& log. dist Solis a Terra		$4, 993967$
Subtrahatur ergo tempus propositum	Febr.	$3^d, 8^h, 3\frac{1}{2}'$
a tempore perihelii — — — —	Febr.	$30, 8, 2$
erit temp. intervallum	$=$	$27^d - 1\frac{1}{2}'$
seu — — —	$=$	$26,9989$ dieb.

Jam ad anomaliam veram inveniendam, quæratur numerus

$$N = \frac{aa}{m\sqrt{b}}.$$

la^2	$=$	8, 693586
$l\sqrt{b}$	$=$	2, 323896
		6, 369690
lm	$=$	5, 434553
lN	$=$	0, 935137
subtr. a l int. temp.		1, 431346
$l(t + \frac{1}{3}t^3 - \frac{2}{5}nt^5)$	$=$	0, 496209

35. Quoniam vero numerus n tantopere est exiguus ut pro nihilo haberi queat, hinc anomalia vera v ex tabula pro motu cometarum parabolico ad finem hujus opusculi annexa inveniri poterit. Prodibit autem interpolatione adhibita.

v	$=$	$117°,27',24''$
Perih. a ☊	—	$151, 25, 52$
Elongatio Cometæ a ☊	—	$33°,58',28''$

Ex anomalia vera v reperitur distantia cometæ a sole SC $= f$. Cum enim sit $f = \frac{ab}{a + (b-a)\cos v}$ ob $b = 2a$ erit

$f = \frac{2a}{1 + \cos v} = \frac{a}{(\cos \frac{1}{2} v)^2}$. Cum ergo sit

$\frac{1}{2} v$	$=$	58°,43′,42
erit $l \cos \frac{1}{2} v$	$=$	9, 715250
2 $l \cos \frac{1}{2} v$	$=$	9, 430500
a $l a$	$=$	4, 346783
Erit $l f$	$=$	4, 916283

Hæc igitur ad figuram primam referendo. Cum sit:

Locus Nodi ☊	—	1ˢ,15°,46′,6″
Long. Terræ	—	4, 14, 26, 13
Erit angulus TSN $= s =$		88°,40,7″

36. Ex determinationibus, quas supra in explicatione hujus figuræ invenimus, dum posuimus: ST $= c$, SC $= f$; CSN $= \Phi$, TSN $= s$, inclinationem orbitæ ad Eclipticam $= i$; atque ang. SNT $= n$, & latitudinem cometæ geocentricam $= p$; ex supra inquam inventis formulis colligitur fore: $\tan n = \frac{c \sin s - f \sin \Phi \cos i}{f \cos \Phi - \cos s}$, &

$$\frac{\cos n}{\tan p} = \frac{f \cos \Phi - c \cos s}{f \sin \Phi \sin i}.$$

Quæ formulæ fortaſſe commodiores ſunt ad locum geocentricum ex heliocentrico inveniendum, quam modus trigonometricus uſitatus. Cum igitur ſit

$l c =$	4, 993967
$l f =$	4, 916283
$i =$	47°,10′,53″
$s =$	88°,40, 7″
$\phi =$	33°,58′,28″

calculus ſequenti modo commodiſſime inſtituetur.

	$l\, c =$	4, 993967
add	l ſin $s =$	9, 999882
	l coſ $s =$	8, 366080
	$l\, c$ ſin $s =$	4, 993849
	$l\, c$ coſ $s =$	3, 359967
	$l\, f =$	4, 916283
add.	l ſin $\phi =$	9, 747270
	l coſ $\phi =$	9, 918705
	$l\, f$ coſ $\phi =$	4, 834988
	$l\, f$ ſin $\phi =$	4, 663553
add.	l coſ $i =$	9, 832300
	l ſin $i =$	9, 865400
	$l\, f$ ſin ϕ coſ $i =$	4, 495853
	$l\, f$ ſin ϕ ſin $i =$	4, 528953
	c ſin $s =$	98594
	f ſin ϕ coſ $i =$	31322
	Num: $=$	67272
	f coſ $\phi =$	68389
	c coſ $s =$	2291

Denom.

Denom. =	66098
l Num: =	4, 827834
l Den. =	4, 820188
l tang n =	10, 007646
Ergo n =	45, 30, 15
add. s =	88, 40, 7
	134, 10, 22
STN = t =	45, 49, 38
seu t =	1ˢ, 15, 49, 38
Locus solis =	10, 14, 26, 13
Longitudo Cometæ =	0ˢ, 0, 15, 51
l cos n =	9, 845630
l f sin ϕ sin i =	4, 528953
	4, 374583
l Denom. =	4, 820188
l tang p =	9, 554395
Latitudo Geocentr.	19°, 43′, 5″

Longitudo ergo per calculum inventa 2′, 35″, & latitudo tantum 12″ ab observationibus discrepant. Hujus modi autem tantilli errores cum adeo in planetis majores quandoque occurrere soleant, merito condonantur.

37. Determinemus jam temporis momenta, quibus cometa per suos nodos transierit. Quorum locorum cum constent anomaliæ veræ, calculus ita se habebit.

	Pro ☊	Pro ☋	
Anomalia vera v =	151, 25,52	28, 34, 8	
$\frac{1}{2} v$ =	75, 42,55	14, 17, 5	
$l\,t$ =	0, 594110	9, 405870	
$l\,t^3$ =	1, 782330	8, 217610	
$l\,3$ =	0, 477121	0, 477121	
$l\frac{1}{3}t^3$ =	1, 305209	7, 740489	
t =	3, 9274	0, 25461	
$\frac{1}{3} t^3$ =	20, 1934	0, 00550	
S =	24, 1208	0, 26011	
lS =	1, 382392	9, 415157	
lN =	0, 935137	0, 935137	
	2, 313529	0, 350294	
Temp. a Perihelio	207, 744	2, 2402	
ſeu in diebus:	$207^d, 17^h, 50$	$2^d, 5^h, 45'$	
Perihelium accidit A.1743. Aug.	214, 8′, 2	1, 8, 2	Mart. 1744
A. 1743. Aug.	6, 14, 12	3, 13, 47	Mart. 1744

Hinc cognoſcimus cometam per ſuum nodum aſcendentem jam tranſiiſſe anno 1743 Menſis Auguſti die 7mo mane: Per nodum autem deſcendentem tranſiit A. 1744 Mart. d. 3, 13^h, 47′ tempore Pariſino medio, quæ ſatis congruunt cum iis, quæ ſupra jam ex orbita cometæ minus exacte cognita derivavimus. Ex obſervationibus autem Caſſinianis quilibet facile concludet, cometam hoc circiter tempore eclipticam trajicere debuiſſe, propterea quod ejus latitudo a Febr. d. 25. usque ad 29mum jam octo gradibus decreviſſet, hocque poſteriori tempore adhuc tantum eſſet

6 graduum

6 graduum, quos æstimatione circiter tribus diebus conficere debuisset.

38. Omnino ergo citra eclipticam iste cometa commoratus est 209^d, 23^h, $35'$, quod tempus si a toto tempore periodico, quod plurium est seculorum subtrahatur, relinquetur spatium temporis, quo cometa in hemisphærio australi est versatus; sicque summa inæqualitas inter ejus moram cis & trans eclipticam perspicitur. Cum autem ab Augusto mense anni elapsi in regiones boreales migrasset, tamdiu ab ecliptica boream versus recessit, quoad ejus latitudo heliocentrica maxima seu ipsi inclinationi orbitæ æqualis esset facta, quod evenit dum ab utroque nodo angulo recto esset elongatus. Simili modo post tertium diem Martii austrum versus ab ecliptica recessit, donec a nodo descendente esset angulo 90° remotus. Quando ergo istæ binæ cometæ ab ecliptica maximæ elongationes evenerint investigemus.

Pro maxima Elongatione	boreali	Australi
Anomalia vera Cometæ v =	61°,25′52′	118°,34′,8″
$\frac{1}{2}v$ =	30°,42,56	59, 17′,4
$l\,t$ =	9, 773875	0, 226123
$l\,t^3$ =	9, 321625	0, 678369
$l\,3$ =	0, 477121	0, 477121
$l\frac{1}{3}t^3$ =	8, 844504	0, 201248
$l\,N$ =	0, 935137	0, 935137

lNt =

$l\,N\,t =$	0, 709012	1, 161260
$l\,\frac{1}{3}N\,t^3 =$	9, 779641	1, 136385
$N\,t =$	5, 11696	14, 4964
$\frac{1}{3}N\,t^3 =$	0, 60206	13, 6894
A Perihelio ad Elongationem maximam	5, 71902	28, 1858
	$5^d, 17^h, 15'$	$28^d, 4^h, 27'$
Perihelium erat Mart.	1 , 8 , 2	1, 8 , 2
seu Febr.	30, 8 , 2	
Febr.	24, 14, 47	29, 12,29' Mart.

Cometa ergo maximam habuit latitudinem borealem heliocentricam Mense Februario 24^d, 14^h, $47'$, hinc ab Augusto anni præteriti cometa usque ad hoc tempus ab ecliptica recessit per tempus 202^d, 0^h, $35'$; inde autem ad eclipticam reversus est 7 diebus 23 horis. Deinde postquam nostris oculis se subduxisset ob nimiam declinationem australem, latitudo ejus heliocentrica increvit usque ad Martii diem 29, 12^h, $29'$, quo tempore erat $47°, 10', 53''$. Nunc igitur iterum ad eclipticam accedit, quam tamen non attinget nisi postquam ab aphelio reversus pervenerit ad anomaliam veram $151°, 25', 52''$, quod demum post complura secula eveniet.

39. Quanquam hanc postremam methodum adhibendo tres tantum cometæ observationes sufficiunt ad ejus orbitam determinandam, tamen quoque in hunc finem plures observationes in usum vocari, sicque errores, qui forte in observationes irrepserint inter omnes æqualiter distribui poterunt ut hoc modo orbita eo propius ad veritatem perducatur.

Sumta

ſumta ſcilicet quacunque obſervatione quarta, ex ea computetur tantum ejus elongatio a linea nodorum pro ſingulis tribus aſſumtis hypotheſibus neque enim ejus diſtantia a ſole opus erit. Quo facto ex elementis ope trium priorum obſervationum inventis, pro ſingulis quoque hypotheſibus quæratur tempus, quo cometa a perihelio ad eam anomaliam veram, quæ quartæ obſervationi competere deprehenſa eſt, pervenire deberet, ſeu computetur ad unamquamque hypotheſin, tempus, quod inter primam obſervationem & quartam interjacet, hocque intervallum cum obſervato comparatum, dabit novam æquationem pro erroribus x & y definiendis, ita ut omnino tres habeantur ejusmodi æquationes, ex quibus duplici modo has litteras x & y determinare licebit Quodſi utroque modo iidem pro his litteris valores prodeant, indicio id erit, jam primam determinationem eſſe exactam, ſin autem quæpiam diſcrepantia reſultet, tum pro x & y valores inter utrosque medii aſſumantur, quo aberratio theoriæ ab obſervationibus ſingulis circiter æqualis reddatur. Simili autem modo quinta, ſexta, pluresque obſervationes in ſubſidium vocari poterunt, quarum ope, ſi iſta præcepta obſerventur, vera cometæ orbita ſatis accurate definiri poterit, etiamſi ſingulæ obſervationes non nimis fuerint exactæ. Hac autem correctione orbita cometæ hic jam inventa non indigere videtur, cum omnibus obſervationibus tam prope ſatisfaciat, ut major conſenſus expectari non debeat.

Cometæ

Cometæ, qui Anno 1742 apparuit, loca observata ad tempus astronomicum medium stili veteris & meridianum Londinensem reducta.

Locus observationis	Tempus æquale Londini St. vet. Febr. 1742	Longitudo Cometæ	Latitudo Cometæ bor.
Peckini	$18^d,8^h,57'$	$9^s,12°,24',0''$	$16°, 58',0''$
Peckini	19, 10, 27	9, 13, 35, 0	22 , 54, 0
Peckini	20 , 8, 26	9, 14, 44, 0	28 , 4, 0
Peckini	21 , 9, 11	9, 16 , 2, 0	33 , 33, 0
Parisiis	21, 17, 34	9, 16 , 6 , 5	35 , 9 , 55
Parisiis	22, 17, 34	9, 17, 46, 35	40, 58 , 37
Peckini	23 , 8, 26	9, 19, 32, 0	45 , 9 , 0
Petropoli	25, 10, 0	9, 24, 13, 0	56 , 29, 0
Peckini	27 , 6, 55	10, 3, 6, 0	66 , 22, 0
Petropoli	27, 10, 13	10, 3, 16, 0	66 , 41, 0
Petropoli	28 , 6, 49	10, 9, 3, 0	70 , 30, 0
Peckini	28 , 8, 55	10, 9, 56, 0	70 , 53, 0
Petropoli	28, 15, 8	10, 11, 56, 0	71 , 54, 0
	Mart.		
Peckini	1 , 7 , 39	10, 18, 19, 0	74 , 20, 0
Petropoli	1 , 7 , 57	10, 19, 12, 0	74 , 30, 0

Petropoli

Petropoli	1, 15, 10	10, 22, 13,0	75, 29, 0
Petropoli	2 , 7 , 2	11, 1, 49,0	77, 24, 0
Petropoli	2 , 7, 39	11, 2, 15,0	77, 28, 0
Peckini	2 , 8, 24	11, 2, 20,0	77, 33, 0
Parisiis	2 , 8, 44	11, 2, 30,5	77, 29, 3
Peckini	3 , 7, 39	11, 19,20,0	79, 22, 0
Peckini	4 , 8, 23	0 , 8,35,0	79, 59, 0
Petropoli	4, 14, 34	0, 13,20,0	79, 55, 0
Peckini	5 , 8, 53	0, 26, 57,0	79, 31, 0
Peckini	6 , 8, 23	1, 10, 11,0	78, 23, 0
Petropoli	6, 13, 55	1, 12, 24,0	77, 46, 0
Petropoli	6, 14, 15	1, 12, 44,0	77, 37, 0
Parisiis	7 , 7, 58	1, 19, 44,40	76, 42, 27½
Peckini	7 , 8, 22	1, 20, 27,0	76, 49, 0
Londini	7, 10, 13	1, 20 , 9,0	76, 38, 0
Parisiis	7, 10, 37	1, 20, 34,0	76, 32, 10
Peckini	8 , 0, 42	1, 24, 46,0	75, 53, 0
Peckini	11, 1, 21	2 , 8, 19,0	71 , 5, 0
Londini	11, 8, 33	2 , 8, 32,0	71 , 8, 0
Peckini	12, 2, 6	2, 10, 52,0	69, 41, 0
Peckini	13, 2, 36	2, 12, 53,0	68, 23, 0
Petropoli	15, 6, 45	2, 16, 18,0	65, 20, 0
Petropoli	16, 6, 38	2, 17, 31,0	64, 16, 0
Peckini	17, 0, 59	2, 18, 34,0	63, 32, 0
Peckini	17, 5, 49	2, 18, 56,0	63 , 8, 0
Petropoli	17, 7, 50	2, 18, 52,0	63, 13, 0
Peckini	18, 6, 19	2, 19, 38,0	62, 21, 0

Peckini	19, 6, 43	2, 20,19,0	61, 33, 0
Peckini	20, 7, 8	2, 20,56,0	60, 47, 0
Parisiis	20, 9, 4	2, 21,12,30	60, 42, 40
Peckini	21, 7, 30	2, 22, 9,0	59, 43, 0

Orbita hujus Cometæ determinata prostat
Tom. VII. Miscellan. Berolinensium.

Distantia perihelii a sole a = 73766 , 36
$l\ a$ = 4, 8678584
Semilatus rectum b est ad a ut 192952 ad 100000
Cometa per perihelium transiit Londini M. Jan. $21^d, 21^h, 52'$
Distantia perihelii a nodo ascendente, $30° 32' 55''$
Longitudo nodi ascendentis $6^s\ 16° 8' 55''$
Inclinatio orbitæ ad eclipticam $56° 35' 7''$

TABULA
MOTUS COMETAE IN PARABOLA.

Anom. vera $= v$ & $t =$ tang $\frac{1}{2}v$.

$v.$ An. vera	$t+\frac{1}{3}t^3$ AreaParab.	$l(t+\frac{1}{3}t^3)$ Logar. Ar.	$v.$ An. vera	$t+\frac{1}{3}t^3$ AreaParab.	$l(t+\frac{1}{3}t^3)$ Lo. Ar. Par.
grad. 0	0, 0000000	$-\infty$	grad. 9	0, 0788642	8, 8968799
	87271	∞		88477	461787
1.	0, 0087271	7,9408700	10	0, 0877119	8, 9430586
	87297	3010946		88747	418584
2.	0, 0174568	8,2419646	11.	0, 0965866	8, 9849170
	87351	1752024		89046	382992
3.	0, 0261919	8,4171670	12.	0, 1054912	9, 0232162
	87431	1260937		89374	353183
4.	0, 0349350	8,5432607	13.	0, 1144286	9, 0585345
	87536	971075		89731	327867
5.	0, 0436886	8,6403682	14.	0, 1234017	9, 0913212
	87672	794253		90114	306097
6.	0, 0524558	8,7197935	15.	0, 1324131	9, 1219309
	87831	672339		90530	287215
7	0, 0612389	8,7870274	16.	0, 1414661	9, 1506524
	88019	583237		90976	270079
8.	0, 0700408	8,8453511	17.	0, 1505637	9, 1777203
	88234	515288		91451	255974
9.	0, 0788642	8,8968799	18.	0, 1597088	9, 2033477

 v.

v. An.vera	$t+\frac{1}{2}t^3$ Ar. Parob.	$l(t+\frac{1}{3}t^3)$ Logar. Ar.	v. An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo. Ar. Par.
grad. 18	0, 1597088	9,2033177	grad. 33	0, 3048770	9, 4841247
	91959	243240		103816	145422
19.	0, 1689047	9,2276417	34.	0, 3152586	9, 4986669
	92497	231548		104885	142137
20.	0, 1781544	9,2507965	35.	0, 3257471	9, 5128806
	93068	221149		106068	139059
21.	0, 1874612	9,2729114	36.	0, 3363539	9, 5267965
	93672	211763		107278	136352
22.	0, 1968284	9,2940877	37.	0, 3470817	9, 5404317
	94310	203259		108539	133731
23.	0, 2062594	9,3144136	38.	0, 3579356	9, 5538048
	94982	195525		109852	131283
24.	0, 2157576	9,3339661	39.	0, 3689208	9, 5669331
	95691	188466		111216	128990
25.	0, 2253267	9,3528127	40.	0, 3800424	6, 5798321
	96433	181997		112641	126850
26.	0, 2349700	6,3710124	41.	0, 3913065	9, 5925171
	97212	176060		114118	124843
27.	0, 2446912	9,3886184	42.	0, 4027183	9, 6050014
	98033	170599		115660	122971
28.	0, 2544945	9,4056783	43.	0, 4142843	9, 6172985
	98888	165557		117260	121216
29.	0, 2643833	9,4222340	44.	0, 4260103	9, 6294201
	99784	160895		118926	119577
30.	0, 2743617	9,4383235	45.	0, 4379029	9, 6413778
	100724	156582		120658	118045
31.	0, 2844341	9,4539817	46.	0, 4499687	9, 6531823
	101703	152575		122459	116614
32.	0, 2946044	9,4692392	47.	0, 4622146	9, 6648437
	102726	148855		124331	115276
33.	0, 3048770	9,4841247	48.	0, 4746477	9, 6763713

$v.$ An.vera	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo.Ar.Par.	$v.$ An.vera	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo.Ar.Par.
grad. 48	0, 4746477	9, 6763713	grad. 62.	0, 6731708	9, 8281252
	126281	114036		163373	104142
49.	0, 4872758	9, 6877749	63.	0, 6895081	9, 8385394
	128303	112873		166905	103875
50.	0, 5001061	9, 6990622	64.	0, 7061986	9, 8489269
	130409	111796		170584	103657
51.	0, 5131470	9, 7102418	65.	0, 7232570	9, 8592926
	132601	110800		174422	103493
52.	0, 5264071	9, 7213218	66.	0, 7406992	9, 8696419
	134876	109873		178421	103374
53.	0, 5398947	9, 7323091	67.	0, 7585413	9, 8799793
	137244	109020		182590	103300
54.	0, 5536191	9, 7432111	68.	0, 7768003	9, 8903093
	139707	108235		186942	103279
55.	0, 5675898	9, 7540346	69.	0, 7954945	9, 9006372
	142270	107517		191480	103298
56.	0, 5818168	9, 7647863	70.	0, 8146425	9, 9109670
	144935	106860		196220	103368
57.	0, 5963103	9, 7754723	71.	0, 8342645	9, 9213038
	147708	106266		201166	103479
58.	0, 6110811	9, 7860989	72.	0, 8543811	9, 9316517
	150594	105729		206334	103635
59.	0, 6261405	9, 7966718	73.	0, 8750145	9, 9420152
	153598	105251		211731	103837
60.	0, 6415003	9, 8071960	74.	0, 8961876	9, 9523989
	156725	104827		217378	104084
61.	0, 6571728	9, 8176796	75.	0, 9179254	9, 9628073
	159980	104456		223277	104375
62.	0, 6731708	9, 8281252	76.	0, 9402531	9, 9732448

$v.$ An. vera	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.	$v.$ An. vera	$t + \frac{1}{3}t^3$ Ar. Parab.	$l(t + \frac{1}{3}t^3)$ Lo. Ar. Par.
76.	0,9402531	9,9732448	91.	1,3688601	10,1363592
	229451	104709		368111	115245
77.	0,9631982	9,9837157	92.	1,4056712	10,1478837
	235911	105087		381665	116346
78.	0,9867893	9,9942244	93.	1,4438377	10,1595183
	242677	105512		395965	117500
79.	1,0110570	10,0047756	94.	1,4834342	10,1712683
	249760	105982		411070	118708
80.	1,0360330	10,0153738	95.	1,5245412	10,1831391
	257185	106491		427036	119978
81.	1,0617515	10,0260229	96.	1,5672448	10,1951369
	264969	107052		443914	121302
82.	1,0882484	10,0367281	97.	1,6116362	10,2072671
	273131	107655		461777	122686
83.	1,1155615	10,0474936	98.	1,6578139	10,2195357
	281699	108304		480706	124140
84.	1,1437314	10,0583240	99.	1,7058845	10,2319497
	290689	109001		500758	125650
85.	1,1728003	10,0692241	100.	1,7559603	10,2445147
	300141	109749		522027	127129
86.	1,2028154	10,0801990	101.	1,8081630	0,2572376
	310074	110539		544617	128869
87.	1,2338228	10,0912529	102.	1,8626247	10,2701245
	320519	111378		568605	130604
88.	1,2658747	10,1023907	103.	1,9194852	10,2831849
	331508	112270		594116	132373
89.	1,2990255	10,1136177	104.	1,9788968	10,2964222
	343078	113209		621261	134256
90.	1,3333333	10,1249386	105.	2,0410229	10,3098478
	355268	114206		650185	136192
91.	1,3688601	10,1363592	106.	2,1060414	10,3234670

v. An.vera	$t+\frac{1}{2}t^3$ Ar. Parob.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.	v. An.vera	$t+\frac{1}{3}t^3$ Ar. Parab	$l(t+\frac{1}{3}t^3)$ Lo. Ar. Par.
106.	2,1060414	10,3234670	120.	3, 4641016	10,5395906
	681009	138210		1439648	176840
107.	2,1741423	10,3372880	121.	3, 6080664	10,5572746
	715916	140702		1531254	180508
108.	2,2457339	10,3513582	122.	3, 7611918	10,5753254
	747051	142120		1630816	184339
109.	2,3204390	10,3655702	123.	3, 9242734	10,5937593
	786624	144783		1739121	188323
110.	2,3991014	10,3800485	124.	4, 0981855	10,6125916
	826836	147157		1857152	192478
111.	2,4817850	10,3947642	125.	4, 2839007	10,6318394
	869928	149623		1985965	196806
112.	2,5687778	10,4097265	126.	4, 4824972	10,6515200
	916132	152190		2126822	201322
113.	2,6603910	10,4249455	127.	4, 6951794	10,6716522
	965746	154857		2281094	206031
114.	2,7569656	10,4404312	128.	4, 9232888	10,6922553
	1019103	157641		2450375	210947
115.	2,8588759	10,4561953	129.	5, 1683263	10,7133500
	1076486	160525		2636466	216076
116.	2,9665245	10,4722478	130.	5, 4319729	10,7349576
	1138350	163535		2841418	221434
117.	3,0803595	10,4886013	131.	5, 7161147	10,7571010
	1205088	166665		3067691	227035
118.	3,2008683	10,5052678	132.	6, 0228838	10,7798045
	1277168	169919		3317927	232890
119.	3,3285851	10,5222597	133.	6, 3546765	10,8030935
	1355165	173309		3593349	238885
120.	3,4641016	10,5395906	134.	6, 7140114	10,8269820

v. An.vera	$t+\frac{1}{3}t^3$ AreaParab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.	v. An.vera	$t+\frac{1}{3}t^3$ AreaParab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.
134.	6, 7140114	10,8269820	148.	17, 625464	11, 2461405
	3905592	245558		1, 608797	379350
135.	7, 1045706	10,8515378	149.	19, 234261	11, 2840755
	4246883	252144		1, 824706	393616
136.	7, 5292589	10,8767522	150.	21, 058967	11, 3234371
	4630260	259188		2, 078751	408835
137.	7, 9922849	10,9026710	151.	23, 137718	11, 3643206
	5059412	266572		2, 379354	425102
138.	8, 4982261	10,9293282	152.	25, 517072	11, 4068308
	5541169	274327		2, 737148	442525
139.	9, 0523430	10,9567609	153.	28, 254220	11, 4510833
	6083643	282480		3, 165844	461238
140.	9, 6607073	10,9850089	154.	31, 420064	11, 4972071
	669622	291052		3, 683004	481379
141.	10, 330329	11,0141141	155.	35, 103068	11, 5453450
	739010	300076		4, 311613	503130
142.	11, 069339	11,0441217	156.	39, 414681	11, 5956580
	817891	309590		5, 081856	526683
143.	11, 887230	11,0750807	157.	44, 496537	11, 6483263
	907868	319629		6, 034015	552278
144.	12, 795098	11,1070436	158.	50, 530552	11, 7035541
	1, 010869	330233		7, 222367	580197
145.	13, 805967	11,1400669	159.	57, 752919	11, 7615738
	1, 129269	341451		8, 720981	610774
146.	14, 935236	11,1742120	160.	66, 473900	11, 8226512
	1, 265913	353338		10, 632908	644416
147.	16, 201149	11,2095458	161.	77, 106808	11, 8870928
	1, 424315	365947		13, 102930	681607
148.	17, 625464	11,2461405	162.	90, 209738	11, 9552535

$v.$ An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.	$v.$ An.vera	$t+\frac{1}{3}t^3$ Ar. Parab.	$l(t+\frac{1}{3}t^3)$ Lo.Ar.Par.
162.	90, 209738	11,9552535	171.	696, 50162	12,8429220
	16, 339289	722960		292, 67111	1523501
163.	106, 549027	12,0275495	172.	989, 17273	12,9952721
	20, 64650	769223		484, 0461	1729950
164.	127, 19553	12,1044718	173	1473, 2188	13,1682671
	26, 48042	821340		861, 6103	1999880
165.	153, 67595	12,1866058	174.	2334, 8291	13,3682551
	34, 54088	880526		1693, 0339	2368196
166.	188, 21683	12,2746584	175.	4027, 8630	13,6050747
	45, 93237	948342		3802, 5759	2887114
167.	234, 14920	12,3694926	176.	7830, 4389	13,8937861
	62, 45521	1026849		10737, 522	3749781
168.	296, 60441	12,4721775	177.	18567, 960	14,2687642
	87, 15848	1118855		44167, 208	5287469
169.	383, 76289	12,5840630	178.	62735, 168	14,7975111
	125, 43102	1228203		438917, 01	9028915
170.	509, 19391	12,7068833	179.	501652, 17	15,7004026
	187, 30771	1360387		∞	∞
171.	696, 50162	12,8429220	180.	infinitum.	infinitum.

Nota.

Orbita Cometæ A. 1742. quæ hic ex Volumine VII. Miſcellan. Berol. recenſetur, eſt ea quidem, quæ per correctionem ibi adhibitam prodiit, cum autem ea ſit nimis lubrica atque a vero vehementer ſeducere poſſit, ſi in obſervationes vel minimus error irrepſerit, præſtabit orbitam ibi primo inventam retinuiſſe, donec ea per methodum ultimam hic expoſitam emendetur: Erat ergo.

I. Diſtantia Perihelii a Sole = 75210

II. Semi-latus rectum = 150420

III. Cometa per perih. tranſiit Tempore Lond. medio St. vet. A. 1742. M. Januario 27^d, 4^h, $14'$

IV. Diſtantia Perihelii a nodo aſcendente $31°$, $17'$, $16''$

V. Longitudo helioc. Nodi aſcendentis 6^s, $9°$, $32'$, $7''$

VI. Inclinatio Orbitæ ad Eclipticam. 61, 43, 44

Berolini 1744.

Ære Michaelis.

Tabula I.

Fig: 1.

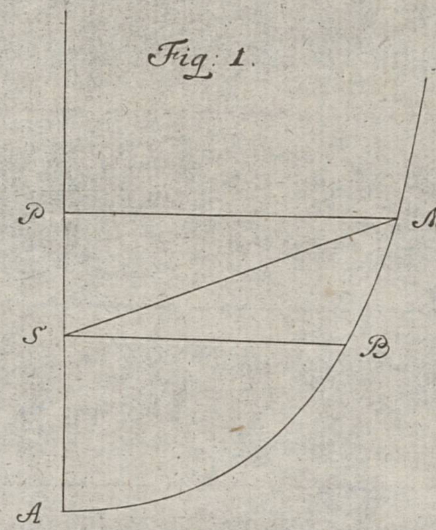


Fig: 2.

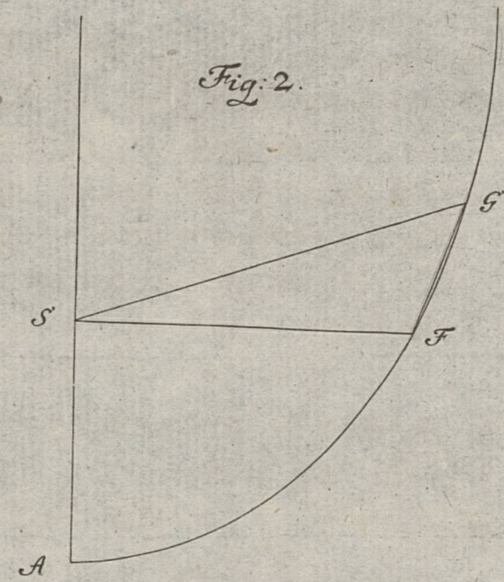


Fig: 3.

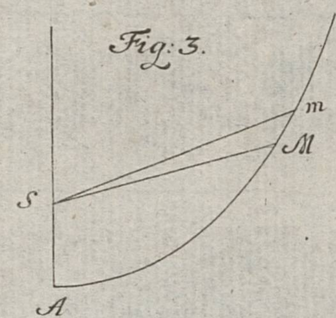


Fig: 4.

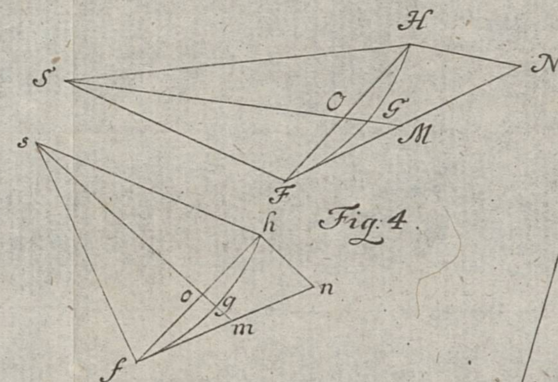


Fig: 6.

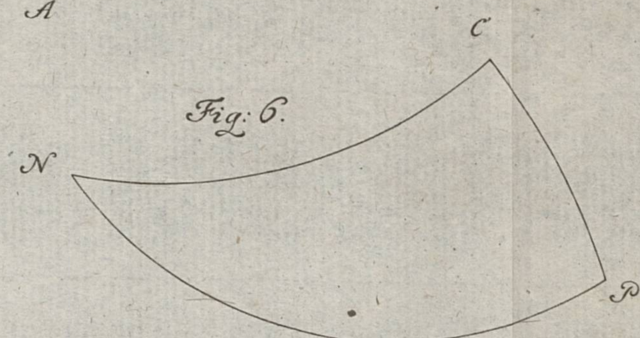
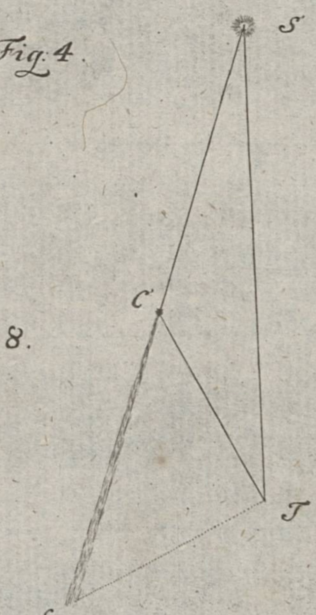


Fig: 8.



F.H.Frisch sc.

Tabula II.

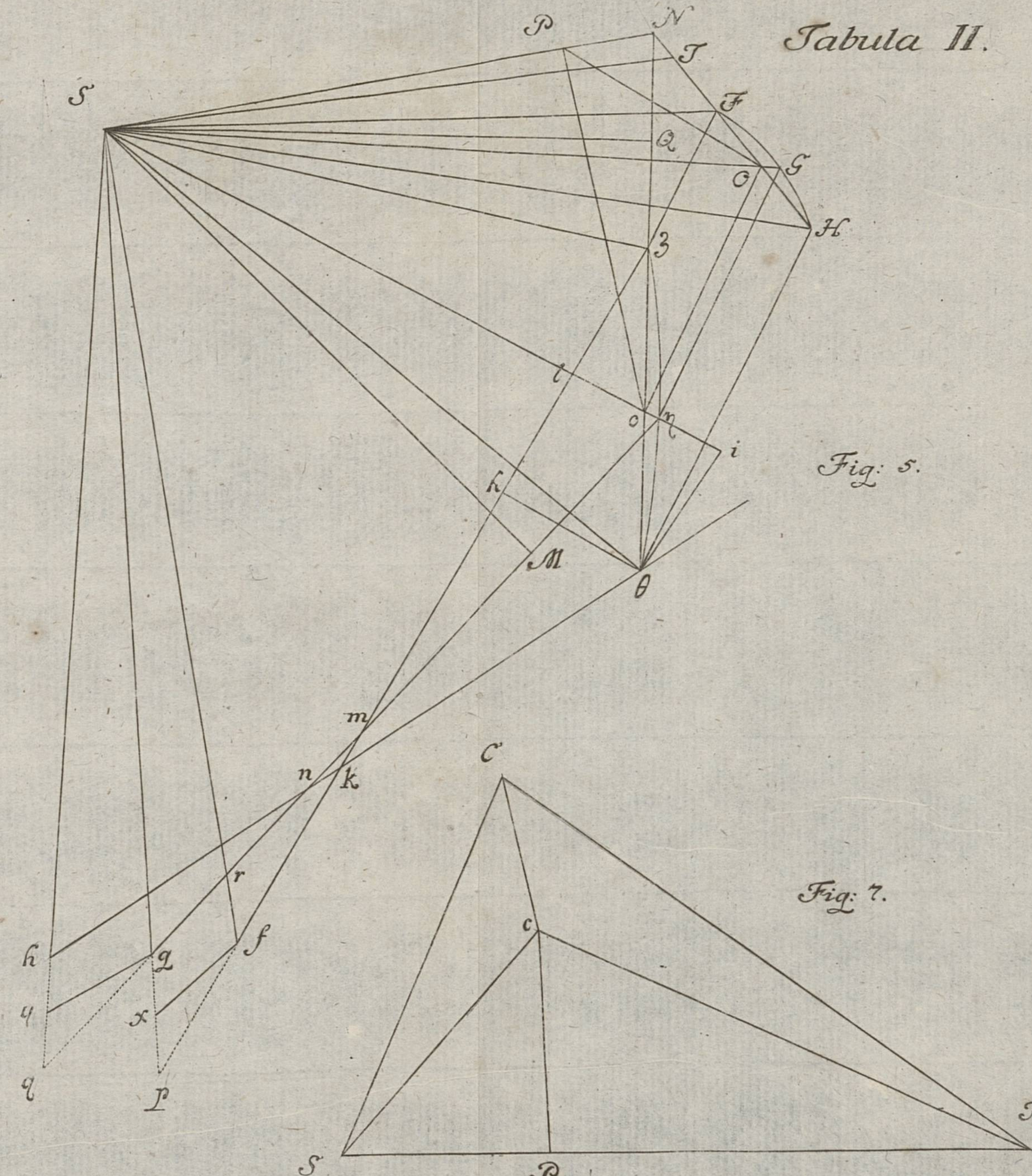


Fig: 5.

Fig: 7.

Tabula III.

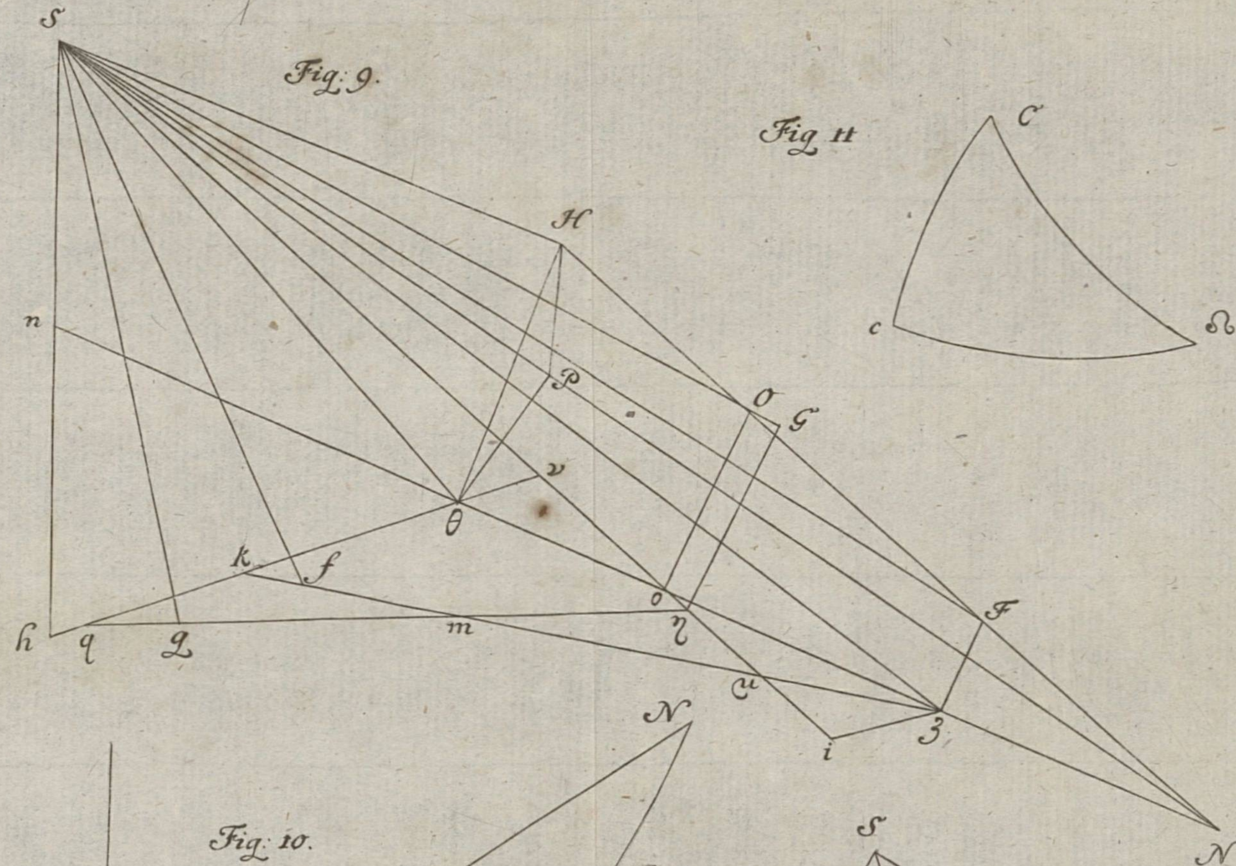


Fig: 9.

Fig. 11

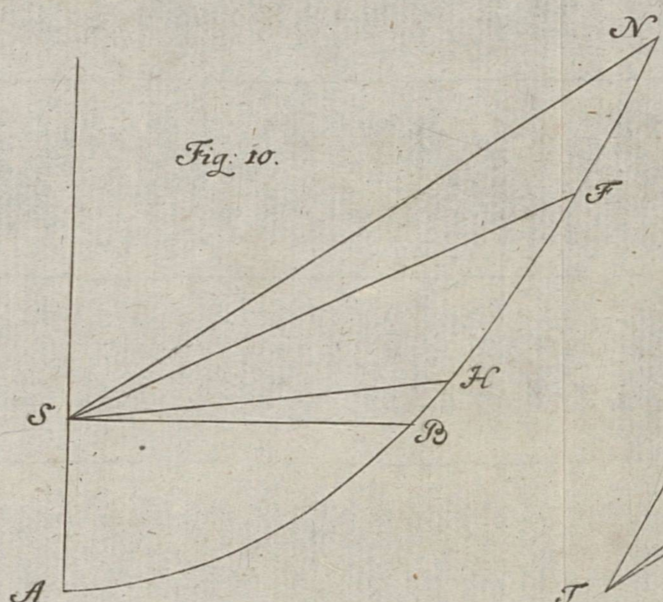


Fig: 10.

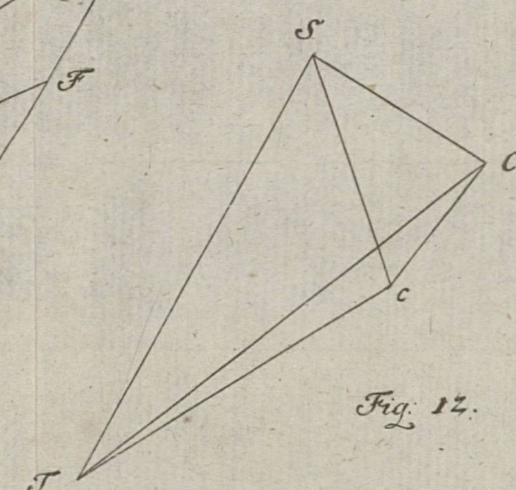


Fig: 12.

Tabula ad Additamentum

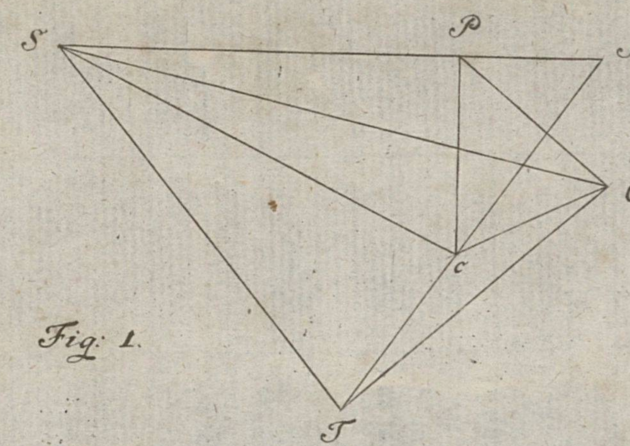
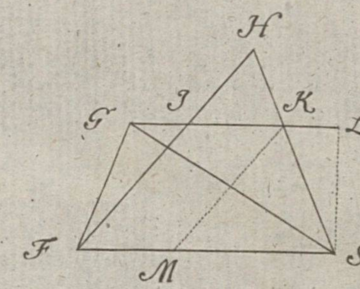


Fig: 1.

Fig: 3.



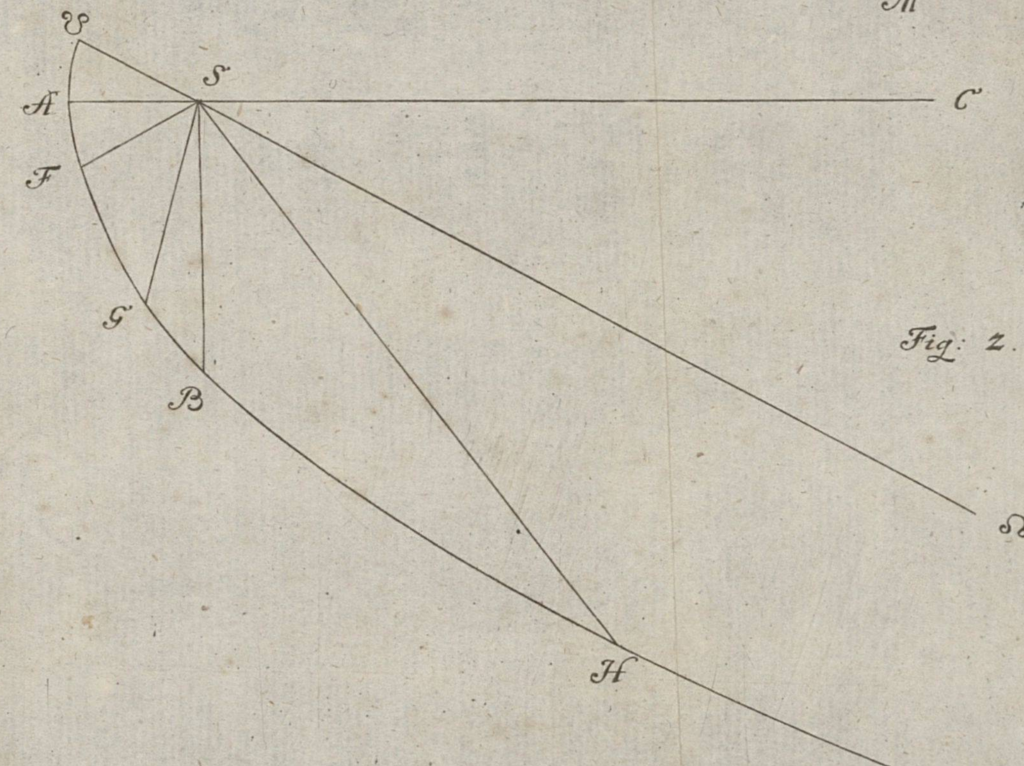


Fig: 2.

inches 4 3 2 1 0

0 1 2 3 4 5 6 7 8 9 10 centimeters

	1	2	3	4	5	6	7	8	9	10	11 (A)	12	13	14	15
L*	39.12	65.43	49.87	44.26	55.56	70.82	63.51	39.92	52.24	97.06	92.02	87.34	82.14	72.06	62.15
a*	13.24	18.11	-4.34	-13.80	9.82	-33.43	34.26	11.81	48.55	-0.40	-0.60	-0.75	-1.06	-1.19	-1.07
b*	15.07	18.72	-22.29	22.85	-24.49	-0.35	59.60	-46.07	18.51	1.13	0.23	0.21	0.43	0.28	0.19
Density										0.04	0.09	0.15	0.22	0.36	0.51

	16 (M)	17	18 (B)	19	20	21	22	23	24	25	26	27	28	29	30	
L*	49.25	38.62	28.86	16.19	8.29	3.44	31.41	72.46	72.95	29.37	54.91	43.96	82.74	52.79	50.87	L*
a*	-0.16	-0.18	0.54	-0.05	-0.81	-0.23	20.98	-24.45	16.83	13.06	-38.91	52.00	3.45	50.88	-27.17	a*
b*	0.01	-0.04	0.60	0.73	0.19	0.49	-19.43	55.93	68.80	-49.49	30.77	30.01	81.29	-12.72	-29.46	b*
Density	0.75	0.98	1.24	1.67	2.04	2.42										

D50 Illuminant, 2 degree observer

Density ⟶

Golden Thread

Colors by Munsell Color Services Lab

Don Williams